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/m170.sp08/handouts170/Epip106/ReviewProbs106

**Pencils and Erasers Only – No Calculators Needed.**

In each of the following problems you are given a function  $f$  and an interval  $[a, b]$ . For each such pair,

- (a) Determine the maximum value of  $f(x)$  on the interval  $[a, b]$ . Announce your result in this format:

$$f_{max} = f(-) = --$$

- (b) Determine the minimum value of  $f(x)$  on the interval  $[a, b]$ . Announce your result in this format:

$$f_{min} = f(-) = --$$

- (c) Determine whether the graph of  $f$  restricted to  $[a, b]$  passes the horizontal-line test (HLT).

- (d) Compute the *Average Rate of Change* (AROC) of  $f(x)$  on the interval  $[a, b]$ .

The results should be announced in “exact algebraic form” without use of approximations.

**Example 1:** Let  $0 < A < B < 2A$  and consider  $f(x) = (2A - x)x$  on the interval  $[-B, B]$ .

As can be seen from a graph,

(a)  $f_{max} = f(A) = A^2$

(b)  $f_{min} = f(-B) = -2AB - B^2$

(c) HLT: no, because, for instance,  $f(B) = f(2A - B)$ .

(d)  $AROC = \frac{f(B) - f(-B)}{B - (-B)} = 2A$

**Example 2:** Let  $f(x) = 4 \sin\left(\frac{x}{3}\right)$  on the interval  $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ .

Sketching a graph will show that

(a)  $f_{max} = f\left(\frac{\pi}{2}\right) = 2$

(c) HLT: yes

(b)  $f_{min} = f\left(\frac{3\pi}{2}\right) = 4$

(d)  $AROC = \frac{2}{\pi}$

These problems give you an opportunity to turn off your calculator and reacquaint yourself with sinusoidal graphs, completing the square, long division, the clock-face trig-function values, the laws of logarithms, and the lore of circles.

- 1 Let  $f(x) = -3x + 2$  on the interval  $[-1, 5]$ .
- 2 Let  $f(x) = 3x^2 - 12x + 15$  on the interval  $[-1, 5]$ .
- 3 Let  $f(x) = 3x^2 + 12x + 15$  on the interval  $[-1, 5]$ .
- 4 Let  $f(x) = 3 - \sqrt{x + 4}$  on the interval  $[-1, 5]$ .
- 5 Let  $f(x) = 3 - \sqrt{36 - x^2}$  on the interval  $[-1, 5]$ .
- 6 Let  $f(x) = 3 - \ln(e^2x)$  on the interval  $[e^{-1}, e^3]$ .
- 7 Let  $f(x) = \ln\left(\frac{1}{x}\right)$  on the interval  $[e^{-1}, e^3]$ .
- 8 Let  $f(x) = -3 \sin(4\pi x)$  on the interval  $\left[-\frac{1}{8}, \frac{1}{3}\right]$ .
- 9 Let  $f(x) = |x - 1|$  on the interval  $[-5, -2]$ .
- 10 Let  $f(x) = \frac{3x - 13}{x - 4}$  on the interval  $[-5, -2]$ .