Pencils and Erasers Only – No Calculators Needed.

In each of the following problems you are given a function \( f \) and an interval \([a, b]\). For each such pair,

(a) Determine the maximum value of \( f(x) \) on the interval \([a, b]\). Announce your result in this format:
\[
 f_{\text{max}} = f(\_\_) = \_
\]

(b) Determine the minimum value of \( f(x) \) on the interval \([a, b]\). Announce your result in this format:
\[
 f_{\text{min}} = f(\_\_) = \_
\]

(c) Determine whether the graph of \( f \) restricted to \([a, b]\) passes the horizontal-line test (HLT).

(d) Compute the Average Rate of Change (AROC) of \( f(x) \) on the interval \([a, b]\).

The results should be announced in “exact algebraic form” without use of approximations.

**Example 1:** Let \( 0 < A < B < 2A \) and consider \( f(x) = (2A - x)x \) on the interval \([-B, B]\).

As can be seen from a graph,

(a) \( f_{\text{max}} = f(A) = A^2 \)

(b) \( f_{\text{min}} = f(-B) = -2AB - B^2 \)

(c) HLT: no, because, for instance, \( f(B) = f(2A - B) \).

(d) \( \text{AROC} = \frac{f(B) - f(-B)}{B - (-B)} = 2A \)

**Example 2:** Let \( f(x) = 4\sin\left(\frac{x}{3}\right) \) on the interval \([\frac{\pi}{2}, \frac{3\pi}{2}]\).

Sketching a graph will show that

(a) \( f_{\text{max}} = f\left(\frac{\pi}{2}\right) = 2 \)

(b) \( f_{\text{min}} = f\left(\frac{3\pi}{2}\right) = 4 \)

(c) HLT: yes

(d) \( \text{AROC} = \frac{2}{\pi} \)
These problems give you an opportunity to turn off your calculator and reacquaint yourself with sinusoidal graphs, completing the square, long division, the clock-face trig-function values, the laws of logarithms, and the lore of circles.

1. Let \( f(x) = -3x + 2 \) on the interval \([-1, 5]\).

2. Let \( f(x) = 3x^2 - 12x + 15 \) on the interval \([-1, 5]\).

3. Let \( f(x) = 3x^2 + 12x + 15 \) on the interval \([-1, 5]\).

4. Let \( f(x) = 3 - \sqrt{x + 4} \) on the interval \([-1, 5]\).

5. Let \( f(x) = 3 - \sqrt{36 - x^2} \) on the interval \([-1, 5]\).

6. Let \( f(x) = 3 - \ln(e^2x) \) on the interval \([e^{-1}, e^3]\).

7. Let \( f(x) = \ln \left( \frac{1}{x} \right) \) on the interval \([e^{-1}, e^3]\).

8. Let \( f(x) = -3 \sin(4\pi x) \) on the interval \([-\frac{1}{8}, \frac{1}{3}]\).

9. Let \( f(x) = |x - 1| \) on the interval \([-5, -2]\).

10. Let \( f(x) = \frac{3x - 13}{x - 4} \) on the interval \([-5, -2]\).