Sun May 2 11:45:58 MDT 2004 Version 2 /m160.sp03/handouts160/t4_160_515/t4_160_515_ANS

1 (a) $f'(x) = \ln(x)$

(b) $f_x = 12x^3 - 6y^3$ and $f_y = -18xy^2 - e^y$

(c) $f_x = \dfrac{2x}{x^2 - y^2}$ and $f_y = -\dfrac{2y}{x^2 - y^2}$

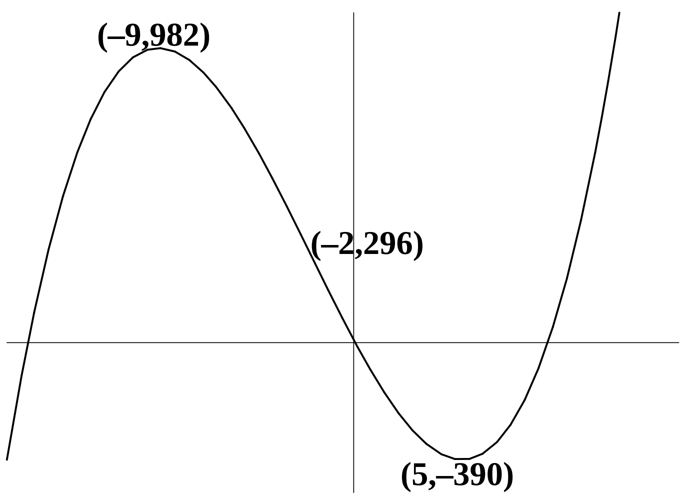
(d) $f_y = -\dfrac{13x}{(4x + y)^2}$

2 (a) 27

(b) $\dfrac{1}{3}(1 - e^{-6})$

(c) $2e^3 + 1$

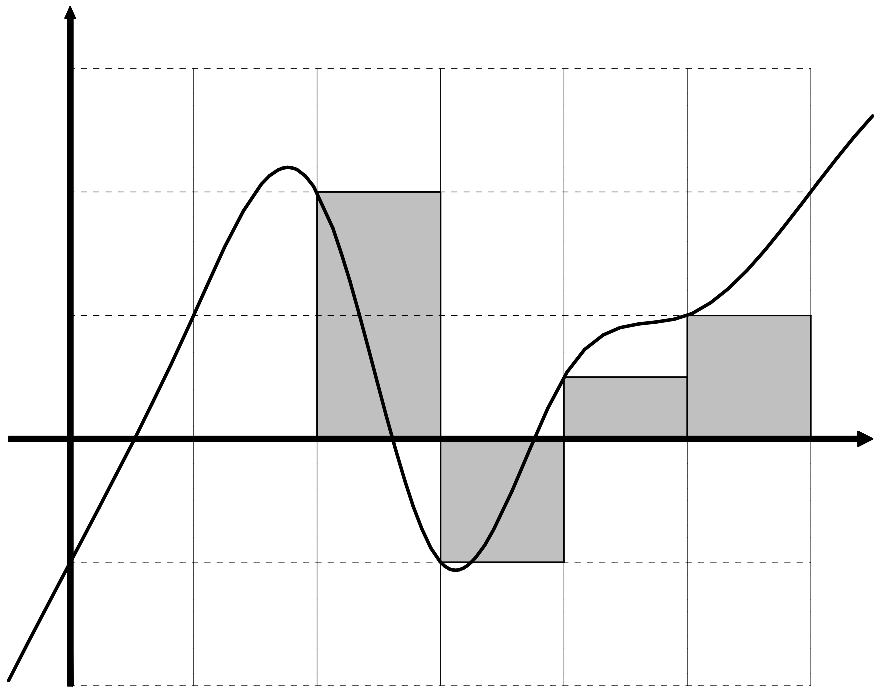
3 $g'(x) = 3(x + 9)(x - 5)$ and $g''(x) = 6(x + 2)$, so the curve has a relative maximum at $(-9, 982)$, an inflection point at $(-2, 296)$ and a relative maximum at $(5, -390)$.



4 g has three x-intercepts and only one y-intercept.

5 $f(x) = 2\,e^{2x} + x^2 - 6$.

6 (a) Here's a picture of the *left sum* L_4 for four equal subdivisions for the integral $\int_2^6 f(x)\,dx$:



(b) $L_4 = 2.5$

7 The graph shows $g' = f$, which can be seen on the left where f has a positive-to-negative x–intercept coinciding with a local maximum for g.

8 $y = 12 - 9(x + 1)$

9 Note that $F''(x) = 6x$, so $F''(-1) < 0$. This means that F is concave downward at and near the point in question. This means that the tangent line lies **ABOVE** the graph of F.

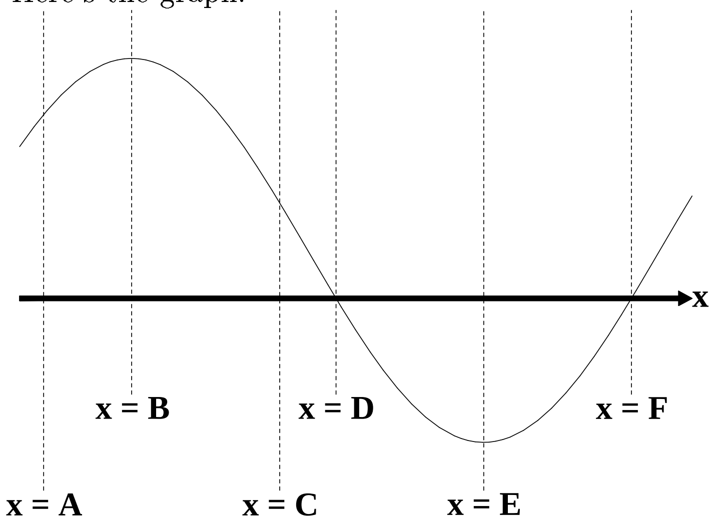
10 For each of the following functions, find all the candidate points at which a maximum or a minimum might occur.

Then go on to classify the candidate points as to whether they are high points, low points, or saddles.

(a) One critical point at $(-2, -2)$. It is a local maximum.

(b) One critical point at $(0, 0)$. Even though both $f_{xx}(0, 0) > 0$ and $f_{yy}(0, 0) > 0$, this critical point yields no extreme.

11 Here's the graph:



$\underline{0}$: $f(F)$	$\underline{+}$: $f'(F)$	$\underline{0}$: $f'(B)$
$\underline{0}$: $f(D)$	$\underline{-}$: $f'(D)$	$\underline{-}$: $f(C)+f(E)$
$\underline{-}$: $f''(B)$	$\underline{+}$: $\int_A^C f(x)\,dx$	$\underline{+}$: $\int_F^D f(x)\,dx$
$\underline{-}$: $\int_C^E f(x)\,dx$	$\underline{+}$: $f''(E)$	$\underline{-}$: $f(B)\times f(E)$
$\underline{+}$: $\int_A^F f(x)\,dx$	$\underline{+}$: $f'(C)\times f'(D)$	$\underline{-}$: $\dfrac{f(F)-f(B)}{F-B}$

12 $\dfrac{dy}{dt} = Ly$, without any steps, immediately yields $y = \mathcal{C}e^{Lt}$. (Just the same way you would invoke the quadratic formula without deriving it from first principles)

$y(0) = 25$ yields $y = 25e^{Lt}$

$y(3) = 52.925$ yields us $L \approx 1/4$, so $y(4) \approx 68$

13 Tabulating the story as it reads,

p	x
2	4800
5	1200

, yields us x as a function of p. The left side of the table gives us $run = 3$ and and the right side gives $rise = -3600$ so that $m = -1200$ and

$$x = 4800 - 1200(p-2), \quad \text{or} \quad x = 7200 - 1200p$$

from which

$$p = 6 - \frac{x}{1200}$$

14

$$R(x) = px = \left(6 - \frac{x}{1200}\right)x = -\frac{1}{1200}x(x - 7200)$$

15 Problem 14 leaves revenue as a function of x rather than of p. The graph of $R(x$ is a frowny parabola with x-intercepts $(0,0)$ and $(7200,0)$. Thus the parabola's axis comes through at $x = 3600$. This means that the maximum revenue happens at $x = 3600$. From problem 13 we get, correspondingly, $p = 6$. This means $R_{max} = 6 \cdot 3600 = 21600$.

Thus revenue is most when the unit price is **$6**. At this price we will sell **3600** units, and take in **$21,600**.