

Pencils, Erasers and “Simple” Calculators Only.

Support your problem solutions with written solution steps. Unsupported correct answers won't get as many points as correct answers supported by solution steps. Unsupported incorrect answers will miss out on any partial credit that's available.

This test has pages 1 – 9. Take a moment to make sure you have them all.

1 Find the indicated derivatives. Simplify:

(a) Find $f'(x)$: $f(x) = x(\ln(x) - 1)$

(b) f_x and f_y : $f(x, y) = 3x^4 - 6xy^3 - e^y$

(c) f_x and f_y : $f(x, y) = \ln(x^2 - y^2)$

(d) Find f_y only: $f(x, y) = \frac{x - 3y}{4x + y}$

2 Evaluate the following integrals exactly – no approximations allowed. Simplify.

(a) $\int_1^2 (3x^2 + 4x^3 + 5) dx$

(b) $\int_0^2 e^{-3x} dx$

(c) $\int_1^{e^3} \ln(x) dx$ Your simplified answers in problem 1 may be of help.

- 3 Use derivatives and factoring to make a rough graph of the function

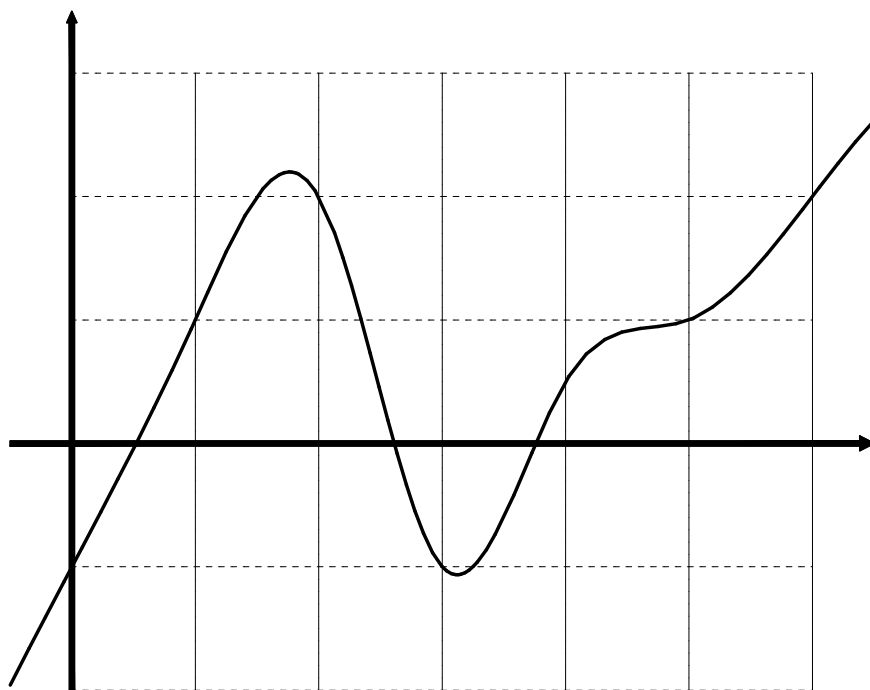
$$g(x) = x^3 + 6x^2 - 135x + 10$$

Label the extreme points and inflection points directly on your graph with their coordinates.

- 4 How many x -intercepts does g have? How many y -intercepts does g have?

5 Find a formula for $f(x)$, given that $f(0) = -4$ and $f'(x) = 4e^{2x} + 2x$.

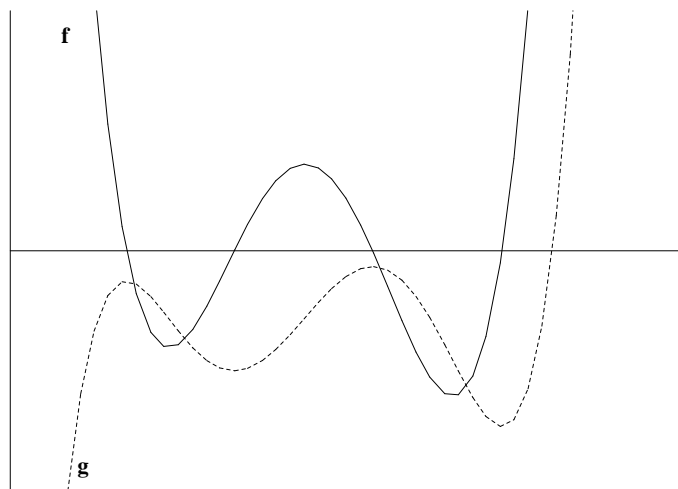
6 Here's a graph of f (the grid spacing is one unit both horizontally and vertically):



(a) Shade in a picture of the *left sum* L_4 for four equal subdivisions for the integral $\int_2^6 f(x) dx$ on this diagram.

(b) Compute the value of this left sum.

7 Consider the graph:



The graph shows functions f and g . One of these is the derivative of the other. In the space to the right of the graph, write an equation which tells which is which. Explain briefly.

8 Find an equation for the line tangent to the graph of $F(x) = x^3 - 12x + 1$ at the point whose x -coordinate is -1 .

9 Explain whether the tangent line in problem 8 lies below the graph of F in the immediate neighborhood of the point of tangency. Use complete sentences in which pronoun referents are clear.

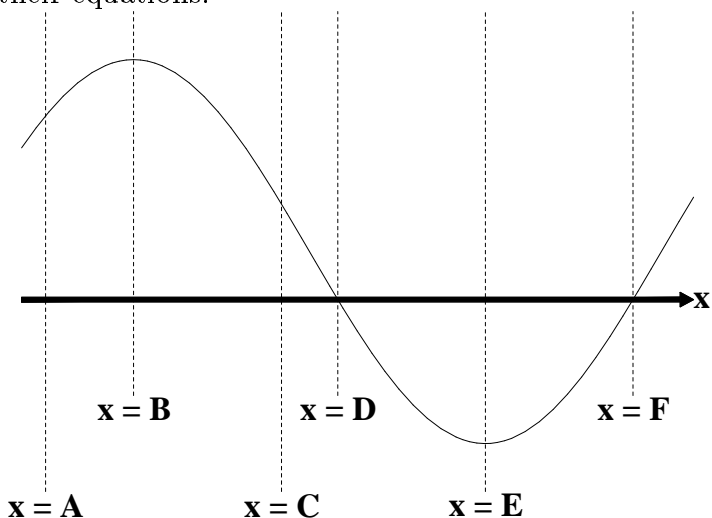
- 10 For each of the following functions, find all the candidate points at which a maximum or a minimum might occur.

Then go on to classify the candidate points as to whether they are high points, low points, or saddles.

(a) $f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4$

(b) $f(x, y) = x^2 - 6xy + 2y^2$

11 Consider this graph which shows the function f and various vertical lines labeled with their equations:



Refer to the graph to decide the *sign* of each of the following expressions. Enter a +, −, or 0 in the adjacent blank according as the expression is positive, negative, or zero.

_____ $f(F)$

_____ $f'(F)$

_____ $f'(B)$

_____ $f(D)$

_____ $f'(D)$

_____ $f(C) + f(E)$

_____ $f''(B)$

_____ $\int_A^C f(x) dx$

_____ $\int_F^D f(x) dx$

_____ $\int_C^E f(x) dx$

_____ $f''(E)$

_____ $f(B) \times f(E)$

_____ $\int_A^F f(x) dx$

_____ $f'(C) \times f'(D)$

_____ $\frac{f(F) - f(B)}{F - B}$

- 12 Consider the differential equation $\frac{dy}{dt} = Ly$, where L is a constant. Find a formula for y as a function of t if $y(0) = 25$ and $y(3) = 52.925$. Use your formula to find $y(4)$.

- 13** Here is a demand-function problem. The weekly Ada-county demand for our innovative remotely actuated bicycle locks behaves as follows: if we set the unit price to \$2, we'll sell 4800, but if we boost it to \$5, we'll only sell 1200.

If x denotes the the number sold per week in Ada county and p denotes the unit price, write a formula for p in terms of x . Assume that p is a linear function of x .

- 14** Continuing from problem 13, give a formula for the weekly-revenue function $R(x)$. Simplify.

- 15** Continuing from problem 14, determine the price at which the revenue is greatest. How many units do we sell at this price? How much money will we take in at this price?