

1 (a) $F'(x) = 6x^2 - 6x$

(b) $f'(x) = -8 \frac{x^2 - 4}{(x^2 + 4)^2}$

(c) $g'(x) = -3e^{-3x}$

(d) $h'(x) = \frac{1}{x}$ (Notice that $h(x) = \ln(x) + \ln(4)$)

2 (a) $k'(x) = 2x e^{x^2+4}$

(b) $f'(x) = \frac{2x}{x^2 + 4}$

(c) $L'(x) = x^3(4 \ln(x) + 1)$

(d) $r(x) = e^{-3x}(1 - 3x)$

3 The dashed line in the picture is the line tangent to the graph of f at the point $(3, f(3))$. The point of tangency is $(3, 4)$ and $m_{tan} = f'(3) = 9$. So the tangent line has an equation $y = 9x - 23$.

4 Here's a sign-change table for $f'(x)$:

---	0	++++	0	----	0	++++
	$x = -1$		$x = 0$		$x = 3$	

The stick-man f graph, beginning on the far left, comes down to a minimum at $x = -1$, then upward to a maximum at $x = 0$, then downward to a minimum at $x = 3$, then ever upward thereafter.

5 Filling in the blanks:

3 $f(x) = 2.5 - e^{-3/4x}$

4 $f(x) = \frac{x}{x-1}$

2 $f(x) = \ln(x)$

5 $f(x) = x^3$

6 $f(x) = e^{-x}$

1 $f(x) = e^x$

- 6 A farmer wants a 200-square-foot rectangular enclosure next to a barn. This means that he doesn't need to fence of the side against the barn. Use calculus to determine the dimensions of the enclosure which requires the least fencing. What is this minimal amount of fencing?

SOLUTION: First draw a picture of the situation: make the barn wall run east-to-west. And nestle the fenced region up to it on the south side of the barn.

Label the unfenced north side of the enclosure with its length x . Label the south side with x also. Label the east and west sides with the length y . Then we know that

$$xy = 200$$

Letting L denote the amount of fencing to be used, we have

$$L = x + 2y$$

The problem is to minimize $L = x + 2y$ subject to the constraints $x > 0$, $y > 0$, and $xy = 200$.

Since $y = \frac{200}{x}$, we can write

$$L = x + 2 \left(\frac{200}{x} \right) = x + \frac{400}{x} = x + 400x^{-1}$$

from which we can get get the derivative

$$L' = 1 - 400x^{-2} = x^{-2}(x^2 - 400) = x^{-2}(x - 20)(x + 20)$$

An L' sign-change chart:

+++	0	---	ND	---	0	+++
	$x = -20$		$x = 0$		$x = 20$	

This shows that $x = 20$ yields the minimum: $L_{min} = L(20) = 40$. For this $y = \frac{200}{20} = 10$.

This says that the farmer will minimize the fencing needed if he makes the side parallel to the barn wall 20 feet long. The sides perpendicular to the barn wall must be 10 feet long. He'll need 40 feet of fencing.