Pencils and Erasers Only – No Calculators Needed.

50 possible

1 Compute and simplify the derivatives of the following:

(a) $f(x) = \sqrt{x^3 - 12x} = (x^3 - 12x)^{1/2}$

$f'(x) = \frac{1}{2}(x^3 - 12x)^{-1/2}(3x^2 - 12) = \frac{3x^2 - 12}{2\sqrt{x^3 - 12x}}$

Chain Rule

(10)

$f(x) = (g(x))^{1/2}$

$f'(x) = \frac{1}{2}(g(x))^{-1/2} g'(x)$

(b) $f(x) = \ln(x^3 - 12x)$

$f'(x) = \frac{1}{(x^3 - 12x)}(3x^2 - 12) = \frac{3x^2 - 12}{x^3 - 12x}$

Chain Rule

(10)

$f(x) = \ln(g(x))$

$f'(x) = \frac{1}{g(x)} g'(x) = \frac{g'(x)}{g(x)}$

Please turn over!

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Chain Rule

2 Find $f'(x)$ and $f''(x)$ if $f(x) = (x+3)e^{2x}$. It's easier if you simplify as you go.

$f'(x) \overset{PR}{=} (1)e^{2x} + (x+3)\left(e^{2x}(2)\right) = e^{2x}\left[1 + 2(x+3)\right]$

$= e^{2x}\left[2x+7\right]$ (10)

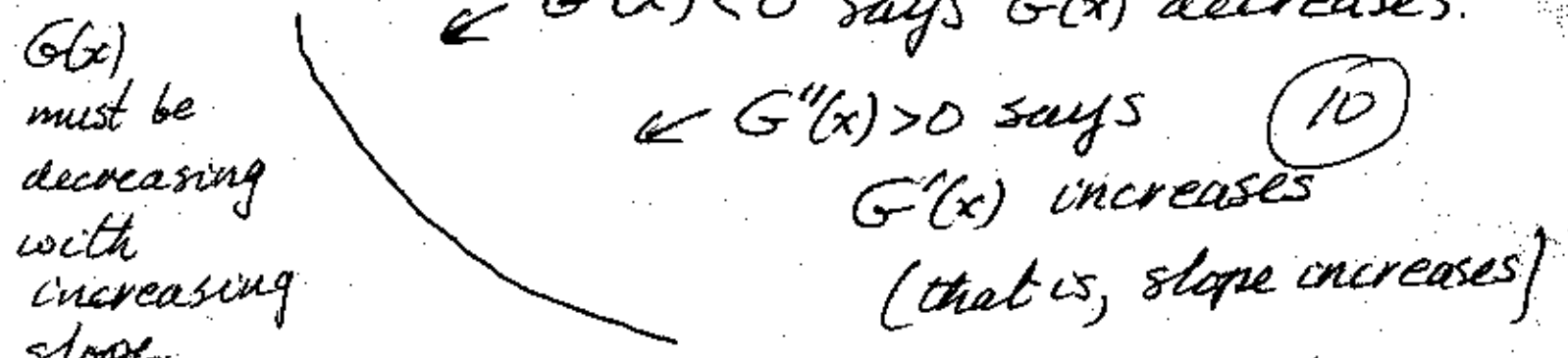
$f''(x) = 2e^{2x}\left[2x+7\right] + e^{2x}\left[2\right] = e^{2x}\left[2(2x+7)+2\right]$

$= e^{2x}\left[4x+16\right]$ (10)

Need chain rule: $(e^{2x})' = e^{2x}(2) = 2e^{2x}$

If $f(x) = e^{g(x)}$, then $f'(x) = (e^{g(x)})g'(x)$

3 Sketch a bit of the graph of a function G for an interval in which $G'(x) < 0$ and $G''(x) > 0$.



← $G'(x) < 0$ says $G(x)$ decreases.

← $G''(x) > 0$ says (10)
$G'(x)$ increases
(that is, slope increases)

$G'(x) < 0$ and $G''(x) > 0$ are to be happening simultaneously.

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