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/m143.sp07/handouts143/RevProbs420/RevProbs420

- 1 Let  $f(x) = \frac{Ax + B}{Cx + D}$ , where  $A$ ,  $B$ ,  $C$ , and  $D$  are constants. Find formulas for the following:

(a)  $f^{-1}(x)$                       (b)  $f(x^{-1})$                       (c)  $f(x)^{-1}$

- 2 Here's a calculator-graphing problem:

Make a graph of  $f$ , where

$$f(x) = (1 - \ln(x))(e^x - 1)$$

for  $1 \leq x \leq 3$ . Sketch your calculator graph on paper. Label the following points on the sketch with their  $x$ - and  $y$ -coordinates (two-decimal accuracy):

- (a) Left endpoint                      (d) Rightmost  $x$ -intercept.  
 (b) Right endpoint                      (e) Maximum point  
 (c) Leftmost  $x$ -intercept                      (f) Minimum point

- 3 Given the table

x	y
2	3
-4	5

find formulas for the following functions.

- (a) A straight-line function  $f$ .  
 (b) An exponential function  $g$ .  
 (c) Circle  $\mathcal{C}$  has its center at the first point of the table. The second point lies on  $\mathcal{C}$ .  
 (d) The graph of  $h$  is a parabola with vertex at the first point of the table. The second point lies on the parabola.  
 (e) The  $x$ -intercepts of  $\mathcal{C}$ .  
 (f) The  $y$ -intercepts of  $\mathcal{C}$ .  
 (g) The  $x$ -intercepts of the graph of  $h$   
 (h) The solutions of  $h(x) = 0$ .

- (i) A formula for  $g^{-1}$
- (j) In terms of base-ten logarithms, the halflife of  $g$ .
- 4 Jones has found a bank that pays 7% annual interest, compounded monthly.
- (a) Suppose Jones makes 4 successive monthly \$100 deposits in an account at this bank. What is the total value of the four deposits immediately after the last deposit?
- (b) How large a single deposit should Jones make now in another account at this bank in order to have it build to \$5000 in two years?
- 5 Write the difference quotient for  $f(x) = \ln(x)$  as the natural logarithm of some quotient.
- 6 Otto has the Ada-county Veeblefetzter franchise. Veeblefetzters are copy protected so that a Veeblefetzter purchased from a franchise in some other county will not fetz in Ada county. This means that Ada-county residents who want a Veeblefetzter have to buy it from Otto. Otto has the following demand data ( $x$  is the weekly sales of Veeblefetzters in Ada county at unit price  $p$ ):

$p$	$x$
50	30
70	24

**Question A:** Write two equations: one expressing  $x$  in terms of  $p$  and one expressing  $p$  in terms of  $x$ . Assume that demand is linear.

**Question B:** Write two formulas for the weekly revenue  $R$ , one expressing  $R$  in terms of  $p$  and one expressing  $R$  in terms of  $x$ .

Otto also has the following data on the weekly total manufacturing cost  $C$  as it relates to the number  $x$  produced and sold each week:

$x$	$C$
9	1080
30	1500

**Question C:** Write formulas for  $C$  in terms of  $x$  and for  $C$  in terms of  $p$ . Assume, in each case, that  $C$  depends linearly on the independent variable.

**Question D:** Now write two formulas for the weekly profit  $P$ , one expressing  $P$  in terms of  $p$  and one expressing  $P$  in terms of  $x$ .

Each of these profit formulas will indicate how to attain the maximum profit. One will give the unit price at which total weekly profit is largest, while the other will yield a number of Veeblefetzters to be made and sold in order to attain a maximum profit.

**Question E:** Find the maximum profit and describe how it may be attained.

7 The function  $f(x) = 5x^2 - 30x - 80$  does not have an inverse, right?

The function  $g$ , given by  $g(x) = 5x^2 - 30x - 80$  for  $x \leq 3$ , does have an inverse, however. Find a formula for  $g^{-1}(x)$ .

8 A table of functions and inverses:

Function	Inverse
$f(x) = \frac{3}{5}x - \frac{7}{10}$	$f^{-1}(x) =$
$f(x) = 3 + 2\sqrt{4 - 5x}$	$f^{-1}(x) =$
$f(x) =$	$f^{-1}(x) = \frac{4 - e^{(x-3)/2}}{5}$
$f(x) = 3 + \frac{2}{4 - 5x}$	$f^{-1}(x) =$
$f(x) = \frac{2 - 3x}{6x - 7}$	$f^{-1}(x) =$
$f(x) =$	$f^{-1}(x) = 125 \left(1 + \frac{3}{x}\right)^3$

9 If  $f(x) = e^x + 4$ , and  $g(x) = \ln(1 - e^{2x})$ , then  $(f \circ g)(x) = ?$

10 If  $g(x) = \frac{e^x - e^{-x}}{2}$ , then  $g(\ln(2)) = ?$

11 If  $f(x) = 8^x$  and  $g(x) = \log_2(x)$ , find  $(f \circ g)(x)$

12 Check out 2.6: 25. Let's add a part (d) which you can do by hand:

25 (d): What are the dimensions of the maximum-area garden plot he can fence off if he has \$600 to spend on fencing?

Make clear which dimension lies along the road.