Example 1: Find a formula for the exponential function $f$ whose graph passes through the points $(0, 7)$ and $(1, 98)$.

We rack up these values in a table:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>1</td>
<td>98</td>
</tr>
</tbody>
</table>

Jumping down one line on the left, we see $x$ adding 1 to itself, while on the left, we see $f(x)$ multiplying itself by 14.

This indicates $f(x) = b \cdot a^x$, where $b$ is the $y$-intercept’s $y$-coordinate and $a$ is the amount by which $f(x)$ multiplies itself for a run of 1.

In this problem, $b = 7$ and $a = 14$.

Check that $f(x) = 7 \cdot 14^x$ is the right thing by verifying that this formula gives $f(0) = 7$ and $f(1) = 98$.

Example 2: Same question as in Example 1, this time with

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>98</td>
</tr>
</tbody>
</table>

Here, when $x$ goes up by 2, $f(x)$ multiplies itself by 49.

This indicates $f(x) = b \cdot a^x$, where $b = 2$, and $a^2 = 49$. This is because $a$ tells us what $f(x)$ multiplies itself by when $x$ adds 1 to itself. Thus $a = 7$.

Check that $f(x) = 2 \cdot 7^x$ is the right thing by verifying that this formula gives $f(0) = 2$ and $f(2) = 98$. 
Example 3: Same question, only this time the $y$-intercept isn’t obvious:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>128</td>
</tr>
<tr>
<td>5</td>
<td>16</td>
</tr>
</tbody>
</table>

Here we see that as $x$ adds 3 to itself, $f(x)$ multiplies itself by $16/128 = 1/8$. Recall that the $a$ in $f(x) = b \cdot a^x$ is the amount by which $f(x)$ multiplies itself as $x$ adds just 1 to itself. So, when $x$ adds three, $f(x)$ has to multiply itself by $a^3 = 1/8$, so $a = 1/2$.

This means that $f(x) = b \cdot \left(\frac{1}{2}\right)^x$.

We can use the points to determine $b$: we must have $f(2) = 128$, and also $f(2) = b \cdot \left(\frac{1}{2}\right)^2$. So

$$128 = b \cdot \left(\frac{1}{4}\right) \quad \text{or} \quad b = 512.$$

Thus $f(x) = 512 \cdot \left(\frac{1}{2}\right)^x$.

An easier way, maybe: since the data start at $x = 2$, we say we want $f(x) = b \cdot a^{(x-2)}$.

Then it’s easy to see that $f(x) = 128 \cdot a^{(x-2)}$, and then that $a = 1/2$ as before. So an alternative formula is

$$f(x) = 128 \cdot \left(\frac{1}{2}\right)^{(x-2)}.$$
Example 4: Let $A(t)$ denote the amount of exponential goo (in tons) $t$ hours after midnight, 3/20-21/07. At 11 PM on 3/20, there was 5 tons of goo, and at 3 PM on 3/21, there was 3.5 tons of goo.

We can tabulate the givens:

<table>
<thead>
<tr>
<th>$t$</th>
<th>$A(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>5</td>
</tr>
<tr>
<td>15</td>
<td>7/2</td>
</tr>
</tbody>
</table>

The run is 16, and over the 16 hours, $A(t)$ multiplies itself by

$$a^{16} = \frac{7/2}{5} = \frac{7}{10}$$

Thus $A(t) = 5 \cdot \left(\frac{7}{10}\right)^{(x+1)/16}$
Exercises: The following exercises are algebra exercises. They do not call for any heavy calculator number crunching. Their answers must not include any decimal points.

1. Find a formula for $y$ as an exponential function of $x$ from the table:

   \[
   \begin{array}{c|c}
   x & y \\
   \hline
   0 & 1 \\
   2 & 9 \\
   \end{array}
   \]

2. Find a formula for $y$ as an exponential function of $x$ from the table:

   \[
   \begin{array}{c|c}
   x & y \\
   \hline
   0 & 5 \\
   2 & 9 \\
   \end{array}
   \]

3. Find a formula for exponential function $f(x)$ from the table:

   \[
   \begin{array}{c|c}
   x & f(x) \\
   \hline
   3 & 5 \\
   10 & 9 \\
   \end{array}
   \]

4. Find a formula for the exponential function whose graph passes through the points $(2, 9)$ and $(5, 7)$.

5. Find the value of $f(2)$ if $f$ is an exponential function whose graph passes through the points $(-2, 4/9)$ and $(6, 1/4)$.

6. At noon a sample of 50,000 little noxious bacteria was moved to a lab petri dish. By 4 PM, they had been fruitful and multiplied their numbers to 170,000.

   Find a formula for $N(t)$, the number of bacteria in the sample $t$ hours after noon.