

3 Let $P(x) = 3x^5 - 14x^4 - 14x^3 + 36x^2 + 43x + 10$.

(a) The “Rat Can List”:

$$\pm 1 \quad \pm 2 \quad \pm 5 \quad \pm 10 \quad \pm \frac{1}{3} \quad \pm \frac{2}{3} \quad \pm \frac{5}{3} \quad \pm \frac{10}{3}$$

(b) To show that $x = -1$ is a zero of multiplicity two for $P(x)$, we do two successive synthetic divisions:

$$\begin{array}{r|rrrrrr} -1 & 3 & -14 & -14 & 36 & 43 & 10 \\ & & -3 & 17 & -3 & -33 & -10 \\ \hline & 3 & -17 & 3 & 33 & 10 & 0 \end{array}$$

which shows $x = -1$ is a zero. And then with the reduced polynomial:

$$\begin{array}{r|rrrrr} -1 & 3 & -17 & 3 & 33 & 10 \\ & & -3 & 20 & -23 & -10 \\ \hline & 3 & -20 & 23 & 10 & 0 \end{array}$$

so $x = -1$ is of multiplicity at least two. One more:

$$\begin{array}{r|rrrr} -1 & 3 & -20 & 23 & 10 \\ & & -3 & 23 & -46 \\ \hline & 3 & -23 & 46 & -36 \end{array}$$

which shows $x = -1$ doesn't have multiplicity three.