

Fri Oct 12 10:18:57 MDT 2007

/m143.007/handouts143/qa12/qa12.143

Pencils and Erasers Only - No Calculators Needed.

45 possible

1. Make a no-calculator tickmark-free graph of $f(x) = 4x^2 - 20x - 75$. Label salient points of your graph with their coordinates. That is, label the vertex and any intercepts with their exact coordinates.

Method A

$$x_{\text{vertex}} = -\frac{b}{2a} = -\frac{(-20)}{2(4)} = \frac{5}{2}$$

$$\frac{5}{2} \left| \begin{array}{ccc} 4 & -20 & -75 \\ & 10 & -25 \\ \hline 4 & -10 & -100 \end{array} \right.$$

$$-100 = y_{\text{vertex}} = f(x_{\text{vertex}})$$

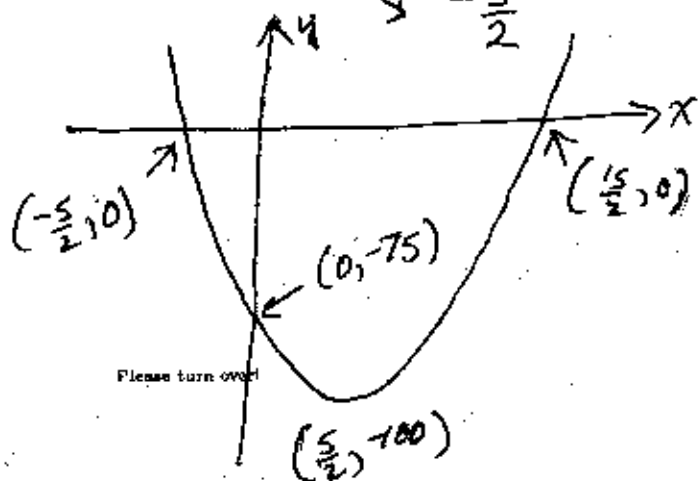
$$\therefore f(x) = 4\left(x - \frac{5}{2}\right)^2 - 100$$

x-intercepts

$$4\left(x - \frac{5}{2}\right)^2 = 100$$

$$\left(x - \frac{5}{2}\right)^2 = 25; \quad x - \frac{5}{2} = \pm 5$$

$$x = \frac{5}{2} \pm 5 \rightarrow \begin{array}{l} + \rightarrow \frac{15}{2} \\ - \rightarrow -\frac{5}{2} \end{array}$$



Please turn over

Method B

$$f(x) = 4\left[x^2 - 5x + \frac{25}{4} - \frac{25}{4}\right] - 75$$

$$= 4\left[\left(x - \frac{5}{2}\right)^2 - \frac{25}{4}\right] - 75$$

$$= 4\left(x - \frac{5}{2}\right)^2 - 25 - 75$$

$$= 4\left(x - \frac{5}{2}\right)^2 - 100$$

① Shape	5	"^" 1 pt
② y-int	5	"-75" 1 pt.
③ x-ints	5	
④ vertex	5	No y 2 pts

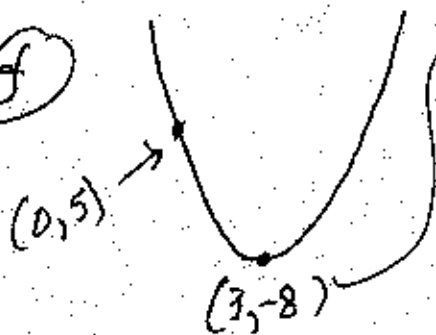
TOTAL 20

20

- 2 The quadratic functions f and g share a y -intercept at $(0, 5)$. The vertex of the graph of f is at $(3, -8)$, while the vertex of the graph of g is at $(3, 6)$. Find formulas for $f(x)$ and $g(x)$.

$$f(6) + g(6) = \underline{10}$$

(f)



$$f(x) = a(x-3)^2 - 8$$

Descartes finds value of a :

$$5 = f(0) = a(0-3)^2 - 8$$

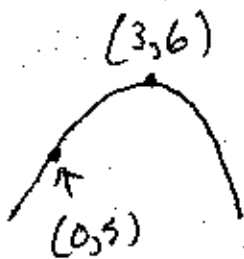
$$5 = a(9) - 8$$

$$13 = 9a \quad \text{so} \quad a = \frac{13}{9}$$

$$\therefore f(x) = \frac{13}{9}(x-3)^2 - 8$$

(10)

(g)



$$g(x) = a(x-3)^2 + 6$$

$$5 = g(0) = a(0-3)^2 + 6$$

$$5 = 9a + 6 \quad \text{so} \quad 9a = -1 \quad \text{or} \quad a = -\frac{1}{9}$$

$$\therefore g(x) = -\frac{1}{9}(x-3)^2 + 6$$

(10)

$$\text{Now } f(6) = \frac{13}{9}(6-3)^2 - 8 = \frac{13}{9}(3)^2 - 8 = \frac{13}{9}(9) - 8 = 13 - 8 = \underline{\underline{5}}$$

$$g(6) = -\frac{1}{9}(6-3)^2 + 6 = -\frac{1}{9}(3)^2 + 6 = -\frac{1}{9}(9) + 6 = -1 + 6 = \underline{\underline{5}}$$

$$\therefore f(6) + g(6) = 5 + 5 = 10$$