1. Make a no-calculator tickmark-free graph of \( f(x) = 4x^2 - 20x - 75 \). Label salient points of your graph with their coordinates. That is, label the vertex and any intercepts with their exact coordinates.

Method A

\[ x_{\text{vertex}} = \frac{-b}{2a} = -\frac{(-20)}{2(4)} = \frac{5}{2} \]

\[ \frac{5}{2} \left[ \frac{1}{4} - \frac{20}{10} - \frac{75}{10} \right] = y_{\text{vertex}} = f(x_{\text{vertex}}) = 4 \left( x - \frac{5}{2} \right)^2 - 100 \]

\[ f(x) = 4 \left( x - \frac{5}{2} \right)^2 - 100 \]

- \( x \)-intercepts
  
  \[ 4 \left( x - \frac{5}{2} \right)^2 = 100 \]
  
  \[ (x - \frac{5}{2})^2 = 25 \]
  
  \[ x - \frac{5}{2} = \pm 5 \]
  
  \[ x = \frac{5}{2} \pm 5 \]
  
  \[ x = \frac{5}{2} \pm 5 \]

Method B

\[ f(x) = 4 \left[ x^2 - 5x + \frac{25}{4} - \frac{25}{4} \right] - 75 \]

\[ = 4 \left[ (x - \frac{5}{2})^2 - \frac{25}{4} \right] - 75 \]

\[ = 4 \left( x - \frac{5}{2} \right)^2 - 25 - 75 \]

\[ f(x) = 4 \left( x - \frac{5}{2} \right)^2 - 100 \]

- Shape 5 "\( \wedge \)" 1 pt
- \( y \)-int 5 "75" 1 pt
- \( x \)-ints 5 No y 2 pts
- Vertex 5 Total 20

\[ \left( \frac{5}{2}, 0 \right) \]

Please turn over.
2. The quadratic functions \( f \) and \( g \) share a \( y \)-intercept at \((0, 5)\). The vertex of the graph of \( f \) is at \((3, -8)\), while the vertex of the graph of \( g \) is at \((3, 6)\). Find formulas for \( f(x) \) and \( g(x) \).

\[
f(6) + g(6) = 10
\]

\[f(x) = a(x-3)^2 - 8
\]

Descartes finds value of \( a \):

\[5 = f(0) = a(0-3)^2 - 8
\]
\[5 = a(9) - 8
\]
\[13 = 9a \quad \text{so} \quad a = \frac{13}{9}
\]

\[f(x) = \frac{13}{9}(x-3)^2 - 8
\]

\[g(x) = a(x-3)^2 + 6
\]

\[5 = g(0) = a(0-3)^2 + 6
\]
\[5 = 9a + 6 \quad \text{so} \quad 9a = -1 \quad \text{or} \quad a = -\frac{1}{9}
\]

\[g(x) = -\frac{1}{9}(x-3)^2 + 6
\]

New \( f(6) = \frac{13}{9}(6-3)^2 - 8 = \frac{13}{9}(3)^2 - 8 = \frac{13}{9}(9) - 8 = 13 - 8 = 5
\)

\(g(6) = -\frac{1}{9}(6-3)^2 + 6 = -\frac{1}{9}(3)^2 + 6 = -\frac{1}{9}(9) + 6 = -1 + 6 = 5
\)

\[f(6) + g(6) = 5 + 5 = 10
\]