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/m143.fa06/handouts143/FindInverseC02/FindInvProbC02

**Example A** In a recent assignment, we graphed the function  $f(x) = e^{x-3} + 4$ . This graph passes the *horizontal-line test* (HLT) and so  $f$  must have an inverse. Let's us find a formula for  $f^{-1}(x)$  :

$$\begin{aligned} y &= e^{x-3} + 4 \\ x &= e^{y-3} + 4 \quad (\text{now isolate } y) \\ -e^{y-3} &= -x + 4 \\ e^{y-3} &= x - 4 \\ y - 3 &= \ln(x - 4) \\ y &= 3 + \ln(x - 4) \end{aligned}$$

so that

$$f^{-1}(x) = 3 + \ln(x - 4) = \ln(e^3) + \ln(x - 4) = \ln(e^3(x - 4))$$

**Example B** This is a more abstract kind of setting where we make use of the The Law of Inverses which says that

$$\text{If } g(A) = B, \quad \text{then } A = g^{-1}(B),$$

provided, of course, that  $g^{-1}$  exists.

Suppose that

$$f(x) = 5 - 3Q(2x + 8),$$

where  $Q$  is an invertible function. Here we find the inverse of  $f$  in terms of the inverse of  $Q$  :

$$\begin{aligned} y &= 5 - 3Q(2x + 8) \\ x &= 5 - 3Q(2y + 8) \quad (\text{now isolate } y) \\ 3Q(2y + 8) + x &= 5 \\ 3Q(2y + 8) &= 5 - x \\ Q(2y + 8) &= \frac{5 - x}{3} \end{aligned}$$

$$2y + 8 = Q^{-1}\left(\frac{5 - x}{3}\right) \quad (\text{by the Law of Inverses})$$

$$2y = Q^{-1}\left(\frac{5 - x}{3}\right) - 8$$

$$y = \frac{Q^{-1}\left(\frac{5 - x}{3}\right) - 8}{2}$$

Thus

$$f^{-1}(x) = \frac{Q^{-1}\left(\frac{5 - x}{3}\right) - 8}{2}$$

**Exercises**

1 Find  $f^{-1}$  :

(a)  $f(x) = 2 - e^{x-3}$

(b)  $f(x) = 3e^{4x-1}$

(c)  $f(x) = 3 \ln(4x - 1)$

(d)  $f(x) = 3 - \ln(x + 2)$

(e)  $f(x) = 2 \ln(3 + e^{5x})$

2 Let  $g$  be an invertible function. In each of the following parts, a function  $f$  is given in terms of  $g$ . For each of these functions  $f$ , find  $f^{-1}$  in terms of  $g^{-1}$ .

(a)  $f(x) = 3 - g(x + 2)$

(b)  $f(x) = 2g(4x - 5)$

(c)  $f(x) = \ln(3 + g(e^{2x}))$