Example A In a recent assignment, we graphed the function \( f(x) = e^{x-3} + 4 \). This graph passes the horizontal-line test (HLT) and so \( f \) must have an inverse. Let’s us find a formula for \( f^{-1}(x) \):

\[
\begin{align*}
  y &= e^{x-3} + 4 \\
  x &= e^{y-3} + 4 \quad \text{(now isolate } y) \\
  -e^{y-3} &= -x + 4 \\
  e^{y-3} &= x - 4 \\
  y - 3 &= \ln(x - 4) \\
  y &= 3 + \ln(x - 4)
\end{align*}
\]

so that
\[
f^{-1}(x) = 3 + \ln(x - 4) = \ln(e^3) + \ln(x - 4) = \ln(e^3(x - 4))
\]

Example B This is a more abstract kind of setting where we make use of the The Law of Inverses which says that

If \( g(A) = B \), then \( A = g^{-1}(B) \),

provided, of course, that \( g^{-1} \) exists.

Suppose that
\[
f(x) = 5 - 3Q(2x + 8),
\]

where \( Q \) is an invertible function. Here we find the inverse of \( f \) in terms of the inverse of \( Q \):

\[
\begin{align*}
  y &= 5 - 3Q(2x + 8) \\
  x &= 5 - 3Q(2y + 8) \quad \text{(now isolate } y) \\
  3Q(2y + 8) + x &= 5 \\
  3Q(2y + 8) &= 5 - x \\
  Q(2y + 8) &= \frac{5 - x}{3} \\
  2y + 8 &= Q^{-1}\left(\frac{5 - x}{3}\right) \quad \text{(by the Law of Inverses)} \\
  2y &= Q^{-1}\left(\frac{5 - x}{3}\right) - 8 \\
  y &= \frac{Q^{-1}\left(\frac{5 - x}{3}\right) - 8}{2}
\end{align*}
\]

Thus
\[
f^{-1}(x) = \frac{Q^{-1}\left(\frac{5 - x}{3}\right) - 8}{2}
\]
Exercises

1  Find $f^{-1}$:

(a) $f(x) = 2 - e^{x-3}$
(b) $f(x) = 3e^{4x-1}$
(c) $f(x) = 3 \ln(4x - 1)$
(d) $f(x) = 3 - \ln(x + 2)$
(e) $f(x) = 2 \ln(3 + e^{5x})$

2  Let $g$ be an invertible function. In each of the following parts, a function $f$ is given in terms of $g$. For each of these functions $f$, find $f^{-1}$ in terms of $g^{-1}$.

(a) $f(x) = 3 - g(x + 2)$
(b) $f(x) = 2g(4x - 5)$
(c) $f(x) = \ln(3 + g(e^{2x}))$