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Example 1: Find a formula for the exponential function f whose graph passes through the points $(0, 7)$ and $(1, 98)$.

We rack up these values in a table:

x	$f(x)$
0	7
1	98

Jumping down one line on the left, we see x adding 1 to itself, while on the left, we see $f(x)$ multiplying itself by 14.

This indicates $f(x) = b \cdot a^x$, where b is the y-intercept's y-coordinate and a is the amount by which $f(x)$ multiplies itself for a run of 1.

In this problem, $b = 7$ and $a = 14$.

Check that $f(x) = 7 \cdot 14^x$ is the right thing by verifying that this formula gives $f(0) = 7$ and $f(1) = 98$.

Example 2: Same question as in Example 1, this time with

x	$f(x)$
0	2
2	98

Here, when x goes up by 2, $f(x)$ multiplies itself by 49.

This indicates $f(x) = b \cdot a^x$, where $b = 2$, and $a^2 = 49$. This is because a tells us what $f(x)$ multiplies itself by when x adds 1 to itself. Thus $a = 7$.

Check that $f(x) = 2 \cdot 7^x$ is the right thing by verifying that this formula gives $f(0) = 2$ and $f(2) = 98$.

Example 3: Same question, only this time the y-intercept isn't obvious:

x	$f(x)$
2	128
5	16

Here we see that as x adds 3 to itself, $f(x)$ multiplies itself by $16/128 = 1/8.$

Recall that the a in $f(x) = b \cdot a^x$ is the amount by which $f(x)$ multiplies itself as x adds just 1 to itself. So, when x adds three, $f(x)$ has to multiply itself by $a^3 = 1/8,$ so $a = 1/2.$

This means that $f(x) = b \cdot \left(\frac{1}{2}\right)^x.$

We can use the points to determine b: we must have $f(2) = 128,$ and also $f(2) = b \cdot \left(\frac{1}{2}\right)^2.$ So

$$128 = b \cdot \left(\frac{1}{4}\right) \qquad \text{or} \qquad b = 512.$$

Thus $f(x) = 512 \cdot \left(\frac{1}{2}\right)^x.$

An easier way, maybe: since the data start at $x = 2,$ we say we want $f(x) = b \cdot a^{(x-2)}.$

Then it's easy to see that $f(x) = 128 \cdot a^{(x-2)},$ and then that $a = 1/2$ as before. So an alternative formula is

$$f(x) = 128 \cdot \left(\frac{1}{2}\right)^{(x-2)}$$

Example 4: Let $\boldsymbol{A(t)}$ denote the amount of exponential goo (in tons) $\boldsymbol{t}$ hours after midnight, 3/20-21/07. At 11 PM on 3/20, there was **5** tons of goo, and at 3 PM on 3/21, there was **3.5** tons of goo.

We can tabulate the givens:

$\boldsymbol{t}$	$\boldsymbol{A(t)}$
$\mathbf{-1}$	$\mathbf{5}$
$\mathbf{15}$	$\mathbf{7/2}$

The run is **16,** and over the **16** hours, $\boldsymbol{A(t)}$ multiplies itself by

$$\boldsymbol{a^{16} = \frac{7/2}{5} = \frac{7}{10}}$$

Thus $\boldsymbol{A(t) = 5 \cdot \left(\frac{7}{10}\right)^{(t+1)/16}}$

Exercises: The following exercises are *algebra* exercises. They do not call for any calculator use at all. Their answers must not include any decimal points.

1 Find a formula for y as an exponential function of x from the table:

x	y
0	1
2	16

2 Find a formula for y as an exponential function of x from the table:

x	y
0	5
2	16

3 Find a formula for exponential function $f(x)$ from the table:

x	$f(x)$
3	5
10	9

4 Find a formula for the exponential function whose graph passes through the points $(2, 9)$ and $(5, 7)$.

5 Find the value of $f(14)$ if f is an exponential function whose graph passes through the points $(-2, 5/9)$ and $(6, 1/5)$.

6 At noon a sample of $50,000$ little noxious bacteria was moved to a lab petri dish.

By 4 PM, they had been fruitful and multiplied their numbers to $180,000$.

Find a formula for $N(t)$, the number of bacteria in the sample t hours after noon.