MATH 143 – Review Sheet for Test #3 – 11/18/05

1 This list is not in final form. Like, stuff may yet be added to it.

2 Test #3 is

   Friday
   11/18/05

3 There will be a with-calculator part of this exam. So have some batteries in your calculator. And bring it to the test.

   Compound interest, anyone?

4 The test will cover the material of Assignments #29 – #37 roughly. It will also cover exponential-function even though we will not have had homework on them (check out the recommended problems for assignment #37).

5 Be sure you can do the following:

   (a) Complex-number arithmetic: divide, multiply, add, subtract, conjugate. Also products of polynomials with complex coefficients.

   (b) Use complex numbers to factor the sum of two perfect squares.

   (c) Use the quadratic formula to find complex zeros.

   (d) Given a polynomial with integer coefficients, be able to cook up the famous Rational-Zero-Candidate List.

   (e) Be fluent at synthetic division.

   (f) Find and use the reduced polynomial.

   (g) It’s handy to know about the famous upper-bound criterion.

   (h) Relate the graph of \( f^{-1} \) to the graph of \( f \).

   (i) Given a formula for \( f(x) \), find a formula for \( f^{-1}(x) \):

      (i) \( g(x) = 2 + \frac{3}{x+5} \) has \( g^{-1}(x) = \frac{-5x+13}{x-2} \) Corrected.

      (ii) If \( G \) is a function with an inverse, and \( f(x) = 3 - 2G(5 - 4x) \), then, in terms of \( G^{-1} \), \( f^{-1}(x) = \frac{5}{4} - \frac{1}{4}G^{-1} \left( \frac{3-x}{2} \right) \).
(j) Be able to graph \( g(x) = 2 + \frac{3}{x + 5} \) and its inverse. Show the graph relating properly to asymptotes.

(k) In 1.7, 3.1, and 3.6 we have had to use sign-change charts to help with graphs of factored polynomials and factored rational expressions.

(i) The polynomial \( P(x) = (x - 3)(x - 1)(x + 1)^2 \) has sign chart

<table>
<thead>
<tr>
<th>Sign Chart</th>
<th>x = -1</th>
<th>x = 1</th>
<th>x = 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>+++++</td>
<td>0</td>
<td>+----</td>
<td></td>
</tr>
</tbody>
</table>

(ii) The rational function \( R(x) = \frac{x^2 - 8x + 116}{x^2 - 8x + 15} \) has sign chart

<table>
<thead>
<tr>
<th>Sign Chart</th>
<th>x = 3</th>
<th>x = 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>++++++++</td>
<td>U</td>
<td>+-----</td>
</tr>
</tbody>
</table>

(l) Use of long division and factoring to graph \( R(x) = \frac{x^2 - 8x + 116}{x^2 - 8x + 15} \).

(m) Old, but still-live, business: do function substitutions correctly

(i) I believe the key to this is exemplified by finding \( g(x + h) \) for

\[
g(x) = \frac{x}{x^2 - 3} - x^2
\]

by first writing

\[
g(\quad ) = \left( \frac{\quad }{(\quad )^2 - 3} \right) - (\quad )^2,
\]

and then filling all the blanks with \( x + h \). This will set you on the right path. For this \( g \), \( NQ \) simplifies to

\[
NQ = \frac{3x^2 - 3 + xh}{(x^2 - 3)[(x + h)^2 - 3]} - 2x - h,
\]

doesn’t it?

(ii) In assignment #18 we had \( TI \). A recent Statesman article mentioned \( TI \) indirectly: click here for a picture of a Master Teacher’s black-board work on \( TI \), and where it comes from. Click here for the accompanying article.

(iii) The assignment-#18 answer key is posted.

(iv) On the old, 12/18/02, final exam: problems 7 and 9.

(v) On assignment #18 we had \( NQ, TI, SYM, \) and \( ANT \). Now we have \( QQ \):

\[
QQ = \frac{f(x + 2h) - f(x - 3h)}{5h}.
\]
If you compute and simplify $Q/Q$ for the function
\[ f(x) = 5 - 3x - 2x^2, \]
it simplifies to $-3 - 4x + 2h$. Not 1.

(vi) For $f(x) = 4^x$,
\[ \frac{f(x + h)}{f(x - h)} = 16^h, \]
and, if we let $T_1 I_2$ denote $TI$ for $f(x) = 2^x$ and let $T_4 I_4$ denote $TI$ for $f(x) = 4^x$, then
\[ \frac{T_4 I_4}{T_1 I_2} = 2^x(2^h + 2^{-h}). \]

(n) Draw graphs at least 25 times as big as your routine capital letters.

(o) Distinguish the equation-graph pairs, page 164, line, circle, and parabola problems. Note that we have added on the exponential-function graphs to our ever-growing page-164 list.

(p) Ungraph. That is, suss out the equation of a given graph:

(i) Problem 79, page 103
(ii) Problems 1 and 4 in the old test #1 for 9/27/02.
(iii) Check the recommended problems under assignment #37.

(q) Recognize the page-164 Friendly-Faces List which we enhanced with the upper half of a circle and with exponential-function graphs.

(r) The section-2.5 moves now as applied to exponential functions.

(s) Be sure you know how the graphs of $f$ and $f^{-1}$ are related. In section 4.2 we study a famous inverse-function pair.

(t) Be able to do the algebra to compute an inverse of a function.

6 Some common errors, aka “howlers”:

(a) Getting 1 for $NQ$ or $TI$ when the function is not a straight line parallel to $y = x$.

(b) On assignment #29, many students have done problem 30 using the notorious Square-Root Howler, use of the bogus equations:
\[ \sqrt{A + B} = \sqrt{A} + \sqrt{B} \quad \sqrt{A - B} = \sqrt{A} - \sqrt{B} \]
to do $\sqrt{A - x^2} = \sqrt{4} - \sqrt{x^2} = 2 - \sqrt{x^2}$, which Mama Nature does not LIKE.
(c) Another corporate difficulty shows up when we try to do something like evaluating $-B$ when $B$ is a complicated expression. Here’s an example:

$$A = (3x - y)^2 (2x + 3y)^2$$
$$B = (3x - y)^2 (2x - 3y)^2$$

yields $A - B = 216x^3y - 144x^2y^2 + 24xy^3$.

(d) If $f(x) = \sqrt{9 - x^2}$ and $g(x) = \sqrt{4 - x^2}$, then $(f \circ g)(x) = \sqrt{5 + x^2}$ and $(g \circ f)(x) = \sqrt{x^2 - 5}$.

7 Old, but still-live, business: be able to

(a) **PEMDAS**: Google gets you the Elko Public Schools, and a more-advanced Purplemath discussion. This is important for correctly directing computers (check problem 2, test #1), as well doing algebra correctly.

(b) Add algebraic fractions using the *Least Common Denominator*

(c) Parse a quadratic-in-$x$-and-$y$ equation to see whether it’s a circle.

(d) Add algebraic fractions using the *Least Common Denominator*

(e) Decode **negative exponents** in expressions.

(f) Decode **fractional exponents** in expressions.

8 Purple-page end-of-chapter problems with all the answers BOB!

(A) For Chapter 2, page 244-245: 10, 11, 12

(B) For Chapter 3, page 328: 1-7, 8(b), 9 (except for the calculator part)

(C) For Chapter 4, page 393, we aren’t on the map yet.