1 More Factoring for Sign Charts - Weird Powers

In this section’s problems, we make sign charts of expressions.

(i) The current MATH-147 text (Precalculus, 4/e by Stewart, Redlin, and Watson) has “coldest-power” factoring information in section 1.3 – especially examples 10 and 11, along with problems 79-90.

(ii) Click here for the 6/9/05 quiz answer key, which furnishes on example which goes on to build a sign chart.

(iii) To make a sign chart for $E(x) = 3(x + 4)^2(x - 5)^{-3} - 3(x + 4)^3(x - 5)^{-4}$, we begin by factoring out the “coldest powers” for the common factors $3$, $(x - 5)$, and $(x + 4)$:

$$E(x) = 3(x + 4)^2(x - 5)^{-3} - 3(x + 4)^3(x - 5)^{-4}$$
$$= 3(x - 5)^{-4}[(x + 4)^2(x - 5) - (x + 4)^3]$$
$$= 3(x - 5)^{-4}(x + 4)^2[(x - 5) - (x + 4)]$$
$$= 3(x - 5)^{-4}(x + 4)^2[-9]$$
$$= -27(x - 5)^{-4}(x + 4)^2$$

Note that, once the common factors were taken out, we worked on factoring the expression left between the square braces.

These even-power factors don’t change sign, so for the sign chart we get:

```
----- 0 ------------------------- U -----
x = -4  x = 5
```
(iv) When we consider

\[ F(x) = 12(x - 6)^2(x - 3)^{3/2} + 6(x - 6)^3(x - 3)^{1/2}, \]

we run up against the problem that calculus does not like radicals. So we have to write this in terms of fractional exponents (Precalculus, 4/e by Stewart, Redlin, and Watson section 1.2):

\[ F(x) = 12(x - 6)^2(x - 3)^{3/2} + 6(x - 6)^3(x - 3)^{1/2}. \]

Now we factor out the 6, then the least power of \((x - 6)\), and then the least power of \((x - 3)\):

\[
F(x) = 6(x - 6)^2 \left[ 2(x - 3)^{3/2} + (x - 6)(x - 3)^{1/2} \right] \\
= 6(x - 6)^2(x - 3)^{1/2} \left[ 2(x - 3) + (x - 6) \right] \\
= 6(x - 6)^2(x - 3)^{1/2} \left[ 3x - 12 \right] \\
= 18(x - 6)^2(x - 3)^{1/2}(x - 4)
\]

The \((x - 3)^{1/2}\) and \((x - 6)\) powers never go negative, so the sign chart comes out

\[ \begin{array}{cccc}
    & 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 & \cdots \\
\hline
x = 3 & 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 & \cdots \\
\end{array} \]

2 Problems for Practice

2.1 Cook up sign charts for the following. Bear in mind that this is an algebra situation, a leave-your-calculator-turned-off situation.

(a) \(2(x + 10)^{1/2} + (x + 4)(x + 10)^{-1/2}\)

(b) \(3(x + 4)^2(x + 18)^4 + 4(x + 4)^3(x + 18)^3\)

(c) \((6x)(x^2 - 21)^{4/3} + x^2(4)(x^2 - 21)^{1/3}(2x)\)

(d) \((36 - x^2)^{1/2} + x \left(\frac{1}{2}\right)(36 - x^2)^{-1/2}(-2x)\)
2.2 Cook up sign charts for the following. This is also a leave-your-calculator-turned-off situation. For the trigonometric expressions which are periodic, just do one cycle, or $[0, 2\pi]$, whichever is shorter. Do $[0, 2\pi]$ for non-periodic trigonometric expressions.

(a) $\sin(\theta) - \frac{1}{2}$

(b) $\sin(\theta) + \frac{1}{2}$

(c) $2\sin(\theta) - \sqrt{3}$

(d) $\sin(2\theta) \left( \theta - \frac{\pi}{2} \right)$

2.3 For these no-calculator exponential sign charts, think of transformations of the exponential function ( Precalculus, 4/e by Stewart, Redlin, and Watson, section 2.5).

(a) $e^4 - e^{-2x}$

(b) $e^4 - e^{2x}$