Note on “in terms of”:
Several places in the following you are asked to express a variable quantity $C$, say, in terms of other variable quantities, $N$, $p$, and $Q$, say.
This means you are to supply an equation whose left-hand side consists of $C$ alone, and whose right-hand side is an algebraic expression whose only variables are $N$, $p$, and $Q$.

Examples on expressing the area, $A$, of a circle

(i) in terms of its radius, $r$: $A = \pi r^2$

(ii) in terms of its diameter, $d$: $A = \frac{\pi d^2}{4}$

(iii) in terms of its circumference, $C$: $A = \frac{C^2}{4\pi}$

And the circumference in terms of the area:

$$C = 2\sqrt{\pi A}.$$ 

1 Story Functions

1.1 We are to build a box, a rectangular box.

(a) Give a formula for the volume of this box for dimensions measured in feet.

(b) Give a formula for the surface area of this box.

(c) Now suppose the box is to be made of different materials: the top is to be made of sheet metal costing $6/ft^2$, the sides of fiberglass sheeting costing $2/ft^2$, and the base of a composite costing $4.50/ft^2$. Give a formula for the cost of the materials in the box in terms of the variable dimensions.
1.2 You-Wish Camp lies on a long, straight, east-west beach. Slat Island lies out to sea, \( W \) miles north of You-Wish Camp. Beezer Camp lies \( L \) miles east of You-Wish.

(a) If Louella, the Slat-Island light-house keeper, can swim \( 2 \, \text{mi/hr} \), how long does it take her to swim from the island to You-Wish?

(b) How long would it take her to swim straight from the island to Beezer Camp?

(c) Suppose Louella starts at the island swimming at an angle \( \theta \) to the east of due south, where \( 0 \leq \theta < 90^\circ \). How long does it take for her to reach the beach?

(d) Give a formula for the angle \( \theta \) Louella should use in order to swim directly to Beezer Camp.

(e) Suppose that, after her swim to the beach, Louella can jog along the beach at \( 6 \, \text{mi/hr} \). In terms of \( L \), \( W \), and \( \theta \), how long does it take her to swim from the island at angle \( \theta \) to the beach and then along to Beezer Camp? Assume that her \( \theta \) is at most as big as the value you found in part (d).

1.3 Suppose we need to build a box as in problem 1.1 with the additional requirements that it have a square base, and no top.

(a) Give a formula for the volume of this box for dimensions measured in feet.

(b) Give a formula for the quantity of sheet metal (in \( \text{ft}^2 \)) needed to build this box.

(c) Now suppose the box is to be made of different materials: the sides are to be built of fiberglass sheeting costing \( \$2/\text{ft}^2 \), and the base of a composite costing \( \$4.50/\text{ft}^2 \). Give a formula for the cost of the materials in the box in terms of the variable dimensions.

(d) Write a formula for the quantity of sheet metal in terms of the side length of the base and the volume of the box.
(e) Write a formula for the materials cost of this box in terms of the side length of the base and the volume of the box.

(f) Now write a formula for the materials cost in terms of the volume of the box and the height of the box.

1.4 Raxmire has the exclusive Interocitor franchise for the eleven western states. Raxmire has the following information as to the Interocitor demand in his franchise area: if he sets the selling price at $5,000, he can sell 700 Interocitors each month in his franchise area. He also knows that for each $100 raise in the selling price, he loses 22 monthly sales.

(a) Find a demand formula. That is, assume that the number of monthly sales, \( s \), has a straight-line dependence on the selling price, \( x \). Find a straight-line formula for the number of monthly sales in terms of the selling price.

(b) Find a formula for the monthly revenue in Raxmire’s Interocitor franchise in terms of the selling price of one Interocitor.

(c) At what level should Raxmire set the price of an Interocitor in order to maximize the total monthly revenue?

1.5 Suppose now we need to make an enclosed box as in problem 1.1 with the additional requirement that the roof of the box make an angle \( \theta \) with the horizontal.

(a) Find a formula for the area of the roof. Give your answer in terms of the angle and the variable dimensions of the base of the box.

(b) How much higher is the high side of the roof than the low side? This is answerable in terms of the variables used in part (e).
2 Factoring

2.1 Factor the following:

(a) \(3x^2y^3 - 6xy^2\)
(b) \(3x^2y^3 - 6y^2x^{-3}\)

2.2 Factor the following down to products of constants, degree-one factors, and powers of degree-one factors. This may require radicals or complex numbers.

(a) \(x^2 - 25\)
(b) \(x^2 + 25\)
(c) \(x^3 - 125x\)
(d) \(3x^2(x - 3) + x^3\)
(e) \((x - 3)^3 + 3x(x - 3)^2\)
(f) \(3(x + 5)^2(x + 3)^2 + 2(x + 5)^3(x + 3)\)
(g) \(6x(x^2 + 9)^2(x - 3)^2 + 2(x^2 + 9)^3(x - 3)\)