Exam 1, MATH 515, Spring 09

Problem 1) (10 pts) (a) Give the definition of a base of a topology on a set X.

(b) Show that $\mathcal{B} = \{B_r(v) : \mathbb{Q} \ni r > 0, v \in E\}$ is a base for the topology on a normed vector space.

Problem 2) (10 pts) Show: If U is neighborhood of x and $V \supset U$ then V is a neighborhood of x.

Problem 3) (10 pts.) Explain the relation between *adherent* and *boundary point*. Give some example.

Problem 4) (10 pts) Let $X = \mathbb{R}$ with the ordinary topology and $S = (-1, 1] \cup \{3\}$.

(a) Find an open subset of S in the induced topology, which is not open in $\mathbb{R}$.

(b) Find ∂S and $\overline{S}$. Find all points adherent to S.

Problem 5) (10 pts) Compare $\partial(S \cup T)$ and $\partial S \cup \partial T$. Hint: Let $S = \mathbb{Q}$ and $T = \mathbb{R} \setminus \mathbb{Q}$ in $\mathbb{R}$ with the ordinary topology.

Problem 6) (10 pts) Give the definition of a topology on a set in terms of closed subsets.

Problem 7) (10 pts) Give the definitions of compactness and sequential compactness. How do the two properties compare?

Problem 8) (10 pts) Let $X = \mathbb{R}$ with the discrete topology. Is X compact? Justify your answer.

Problem 9) (10 pts.) Give an example of a metric space, which is bounded but not totally bounded.

Problem 10) (10 pts) Let E be the set of all real bounded sequences $\alpha = \{\alpha_n\}$ with the supremum norm $||\alpha|| := \sup_n |a_n|$. Show that E is a normed vector space.