

**Homework Assignment 6, MATH 515, Spring 09**

**Problem 14) (8 pts)** Show that on every infinite dimensional normed space  $E$  there exists a discontinuous (unbounded) linear functional. **Hint:** Use a Hamel basis  $B \subset E$ ,  $B = \{v_i : i \in I\}$ ,  $I$  some index set, such that each  $v \in E$  has a *unique* representation  $v = \sum_{j=1}^n c_j v_{i_j}$  with  $c_j$  scalars.

**Problem 15) (8 pts)** Let  $E$  be a normed vector space and let  $\lambda \in E'$  such that  $|\lambda| = 1$ . For any  $\varepsilon > 0$  show that there is an  $x_\varepsilon \in E$  with  $|x_\varepsilon| = 1$  and  $\lambda(x_\varepsilon) > 1 - \varepsilon$ . Give an example to show that there need not be an  $x_0 \in E$  such that  $|x_0| = 1$  and  $\lambda(x_0) = 1$ . **Hint:** Consider  $\ell^1$  and  $\ell^\infty$  from Problem 12.

**Problem 16) (8 pts)** page 91, Chapter IV, §6, Exercise 6