Homework Assignment 2, MATH 515, Spring 09

Problem 4) (16 pts) Let E be a vector space. By a **seminorm** on E one means a function $\sigma : E \to \mathbb{R}$ such that $\sigma(x) \geq 0$ for all $x \in E$, $\sigma(x+y) \leq \sigma(x) + \sigma(y)$ and $\sigma(cx) = |c|\sigma(x)$ for all $c \in \mathbb{R}$ and $x, y \in E$.
(a) Let σ_1, σ_2 be seminorms. Show that $\sigma_1 + \sigma_2$ is a seminorm. If λ_1, λ_2 are numbers ≥ 0 show that $\lambda_1\sigma_1 + \lambda_2\sigma_2$ is a seminorm. By induction show that if $\sigma_1, \ldots, \sigma_n$ are seminorms and $\lambda_1, \ldots, \lambda_n$ are numbers ≥ 0 then $\lambda_1\sigma_1 + \ldots + \lambda_n\sigma_n$ is a seminorm.
(b) Let σ_1, σ_2 be seminorms. Show that $\max(\sigma_1, \sigma_2)$ is a seminorm.
(c) Let σ_1 be a norm and σ_2 be a seminorm. Show that $\sigma_1 + \sigma_2$ and $\max(\sigma_1, \sigma_2)$ are norms.
(d) Let σ be seminorm on E. Show that the set of all $x \in E$ such that $\sigma(x) = 0$ is a subspace of E.

Problem 5) (8 pts) page 46, Chapter II, §5, Exercise 8