MATH 513, Fall 2006, Homework Assignment 1

1. Let $X$ be a space, $x \in X$ and $\delta > 0$. Show:

$$D_X(x, \delta) = \{x' \in X | d(x, x') < \delta\}$$ is open.

2. Show for $Y \subset X$

   (a) $cl_X(Y) = int_X(Y) \cup fr_X(Y)$

   (b) $Y \subset X$ is closed $\iff Y = cl_X(Y)$

3. For $\emptyset \neq Y \subset X$ and $x \in X$, let $d(x, Y) := \inf\{d(x, y) | y \in Y\}$.

   Show the following:

   (a) For $x, x' \in X$: $|d(x, Y) - d(x', Y)| \leq d(x, x')$

   (b) $x \in cl_X(Y) \iff$ every neighborhood of $x$ intersects $Y$ nontrivially

   $\iff d(x, Y) = 0$.

   (c) $Y \subset X$ is closed, $x \in X \setminus Y \implies d(x, Y) > 0$.

4. Let $Y \subset X$ be spaces. Then the following holds:

   (a) $U \subset Y$ is open in $Y \iff$ there exists $V \subset X$ open in $X$ with $U = V \cap Y$.

   (b) $F \subset Y$ is closed in $Y \iff$ there is $G \subset X$ closed in $X$ with $G \cap Y = F$.

   (c) If $Y \subset X$ is open (respectively closed) then $U \subset Y$ is open (respectively closed) in $Y \iff U$ is open (respectively) closed in $X$. 