Selected Answers for Practice exam for the final exam, MATH 333, Fall, 2009

1. For $0 < x < 1$: $y(x) = Ce^{\int_{1/2}^{x} \frac{tdt}{\ln(t)}}$, for $x > 1$: $y(x) = Ce^{\int_{3/2}^{x} \frac{tdt}{\ln(t)}}$
2. $P(t) = e^{(t-1)(e^t-1)}$
3. $y = Cx + x(\ln(x))^2$
4. $e^y \cos(x) + ye^{-x} + e^y = c$
5. $y^2 = Cx^3 - x^2$
6. $y(2) \approx 1$
7. $y = e^{-x}(\cos(x) - 2\sin(x) + x)$
8. $y = c_1e^{-x} + c_2e^x + \sqrt{1 - e^{-2x}} - e^x \sin^{-1}(e^{-x})$
9. $x(t) = 3 + \frac{4}{3}e^{-t} - \frac{21}{2}e^{-2t} + \frac{25}{6}e^{-4t}$
10. $y(t) = 1 - 2te^{-t}$
11. $y(t) = \frac{1}{2}(e^{-t} - e^{-3t}) + (e^{-3(t-2)} - e^{-(t-2)})U(t-2) + \frac{1}{2}(e^{-(t-4)} - e^{-3(t-4)})U(t-4)$
12. Calculate $\int_0^2 f(t)dt = \int_0^1 e^{-t}dt - \int_1^2 e^{-st}dt = \frac{(1-e^{-s})^2}{2}$ and $F(s) = \frac{1}{s}\frac{1-e^{-s}}{1+e^{-s}} = \frac{1}{s}\frac{e^{s/2}-e^{-s/2}}{e^{s/2}+e^{-s/2}} = \frac{1}{s}\tanh(\frac{s}{2})$
13. $\mathbf{X} = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-2t} + c_2 \begin{pmatrix} 3 \\ 2 \end{pmatrix} e^{-t} + \begin{pmatrix} \frac{1}{3} \\ \frac{1}{3} \end{pmatrix}$
14. $\mathbf{X} = e^{2t}\left(c_1 \begin{pmatrix} \cos(2t) \\ 2\sin(2t) \end{pmatrix} + c_2 \begin{pmatrix} \sin(2t) \\ -2\cos(2t) \end{pmatrix}\right)$
15. $\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = c_1 \begin{pmatrix} 3 \\ \frac{1-\sqrt{13}}{2} \end{pmatrix} e^{\frac{-1+\sqrt{13}}{2}t} + c_2 \begin{pmatrix} 3 \\ \frac{1+\sqrt{13}}{2} \end{pmatrix} e^{\frac{-1-\sqrt{13}}{2}t}$
16. $\mathbf{X} = c_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t} + \frac{1}{8} \begin{pmatrix} 5e^{3t} + e^{-3t} \\ e^{3t} + 3e^{-3t} \end{pmatrix}$
17. $\mathbf{X} = \frac{1}{8} \begin{pmatrix} 5 + 2t^2 + 2t + 3e^{2t} \\ 3 + 2t^2 - 2t - 3e^{2t} \\ 8e^{4t} - 8e^{3t} \end{pmatrix}$
18. $y = c_1x^{-1} + c_2x - \frac{1}{4}x + \frac{1}{2}x\ln(x)$
19. $y = 1 - 3x^2 - x^4 - \frac{11}{15}x^6 - \ldots$
20. $e^{tA} = \begin{pmatrix} -e^{2t} + 2e^{-t} & 2e^{2t} - 2e^{-t} \\ -e^{2t} + e^{-t} & 2e^{2t} - e^{-t} \end{pmatrix}$ is the fundamental matrix $\phi(t)$ with $\phi(0) = \mathbf{I}$
21. critical points $y = 0, \pm 1$ with asymptotically $y = -1$ stable, $y = 0$ semi-stable and $y = +1$ unstable.