Review for the Final Exam
MATH 333
Fall 2009

This exam is comprehensive and covers the contents from Chapters 1-8 that have been discussed in class (so *not* 6.2, 6.3 and 7.6, and not certain application topics from 1.3, 3.1, 3.2., 3.3, 5.2 and 5.3). In the following: DE=differential equation, IVP=initial value problem.

1. Recognize order, type and linearity/nonlinearity of DEs.
2. Check whether functions or vector functions solve a DE or a system of DEs. Check whether you have a fundamental set of solutions (Wronskians). Check solutions of IVPs.
3. Write systems of DEs or IVPs in matrix form and vice versa. Write higher order DEs respectively IVPs as first order systems respectively system IVPs.
4. Sketch phase portraits of autonomous first order DEs and 2×2 linear systems (including discussion of attractor, repeller, asymptotically stable).
5. Solve first order DEs using separation of variables, linear by integrating factor (variation of parameters), exact DEs and substitutions (*not* Bernoulli DEs).
6. Solve linear DEs using $y = y_p + y_c$. Determine y_c for constant coefficient and Cauchy-Euler equations using the characteristic equation. Find y_p for constant coefficient using undetermined coefficients (you can *choose* superposition or annihilator approach). Find y_p for given y_c using variation of parameters. Use reduction of order to find a second solution if a first solution of the homogeneous DE is given. Solve corresponding IVPs.
7. Solve IVPs using the Laplace transform and its operational properties.
8. Solve linear systems of DEs from $X = X_c + X_p$ with finding X_c from eigenvalues and eigenvectors, or using exponential matrix. Find Y_p using variation of parameters. Compute exponential matrices using power series definition or Laplace transform. Derive fundamental set of solutions from it. Solve system IVPs using these methods.

9. Solve 2×2 systems of DEs by elimination.

10. Find power series solutions (first order terms) of DEs about ordinary points.

11. Solve nonlinear DEs of the form $F(x, y', y'') = 0$ or $F(y, y', y'') = 0$ using substitution.

12. Find numerical solutions of first order DEs using the Euler method.

13. Find eigenvalues and eigenfunctions of boundary value problems.

14. **Application Problems:** population dynamics (including the logistic equation), radioactive decay, falling bodies and air resistance, spring-mass systems, RLC circuits. Set up IVP from the description of the system. Solve the IVP and discuss the solution (includes e. g. knowledge of notions like damped, undamped, forced motion, period, Hooke's law, Newton's law, Kirchhoff's law, voltage drops in relation with capacity, resistance and inductance, charge and current, etc.)

Remarks: 1.) The linear algebra techniques from Appendix II are used in various items above. Even though not explicitly stated above you should know the basics of matrix and vector algebra, be able to calculate determinants, find inverses of matrices, solve linear systems of equations using Gaussian method, find eigenvalues and eigenvectors of matrices.
2.) The integration techniques require that you are fluent with integration by parts and substitution methods to find anti-derivatives.
3.) Laplace transformation requires that you are fluent with partial fraction decomposition.
4.) Finding zeroes of polynomials is involved in practically all methods. Simple algebraic manipulation, the quadratic formula and factorization of quadratic polynomials by factoring the constant term. Quadratic completion is used in several places.
5.) Trigonometric identities like the double angle identity could be necessary.

No calculators, cell-phones, books etc. are allowed on the final exam!

NOTE: You can bring the copies of the two back cover pages *Table of Laplace transforms* and *Review of Differentiation/Brief Table of Integral* for the exam, which have been distributed in class. Only *those* tables are allowed. Additionally you can bring **one handwritten** sheet ($11 \times 8\frac{1}{2}$) of paper with notes from the class. No copies are allowed.

There will be **eight** problems on the test. This gives you an average of 15 minutes per problem. Focus on demonstrating that you have learned the basic techniques to solve DEs. Try to start on all problems. If you don't understand a question ask! Use the Review problems from the book for preparation. Work through the problems from previous tests and quizzes. Rework some of the homework problems and get in touch with me if you have problems that you cannot work out. Get yourself acquainted with formula tables that you can use. Don't waste time on searching the tables or your notes. You should only have to use those to confirm a formula that you are not sure about. Don't fill you note sheet with worked out examples. You will confuse yourself if the example that shows up on the test is a slight variation. Don't get upset when a result looks ugly. It is a matter of the topic that solutions of simple looking DEs can look quite ugly.

Good Luck!