

Practice exam for the final exam, MATH 333, Fall, 2009

1. Solve the differential equation $\frac{dy}{dx} \ln(x) = xy$.
2. Solve the initial value problem $\frac{dP}{dt} + P = Pte^t, P(0) = 1$.
3. Solve the differential equation $xy' - y = 2x \ln(x)$
4. Find an implicit solution of the differential equation

$$(e^y \sin(x) + ye^{-x})dx = (e^y + e^{-x} + e^y \cos(x))dy$$

5. Find an implicit solution of the homogeneous differential equation

$$2xydy - (x^2 + 3y^2)dx = 0$$

for $x > 0$.

6. Use Euler's method with a step size of $h = \frac{1}{2}$ to estimate $y(2)$ for the initial value problem

$$(2x + y)dx + (x + 2y)dy = 0, y(1) = 0.$$

7. Solve the initial value problem

$$y'' + 2y' + 2y = xe^{-x}, y(0) = 1, y'(0) = -1$$

8. Find the general solution of the differential equation:

$$y'' - y = 2(1 - e^{-2x})^{-\frac{1}{2}}, x > 0$$

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9. A 8 lb weight stretches a spring 4 ft. This spring-mass system resides in a medium offering a resistance to the motion equal to 1.5 times the instantaneous velocity, and an external force is given by $f(t) = 6 + e^{-t}$ (in lb) is being applied. If the weight is released at a position 2 ft above its equilibrium position with downward velocity of 3 ft/s, find its position relative to the equilibrium after 2 s have elapsed.
10. Solve the equation $y(t) + 2 \int_0^t y(t-z) \cos(z) dz = 1$.
11. Solve the initial value problem $y' + 4y + 3 \int_0^t y(\tau) d\tau = f(t)$ with

$$f(t) = \begin{cases} 1 & \text{for } 0 \leq t < 2 \\ -1 & \text{for } 2 \leq t < 4 \\ 0 & \text{for } t \geq 4 \end{cases}$$

and $y(0) = 0$.

12. Given

$$f(t) = \begin{cases} 1 & \text{for } 0 \leq t < 1 \\ -1 & \text{for } 1 \leq t < 2 \end{cases}$$

and $f(t+2) = f(t)$, show that

$$\mathcal{L}\{f(t)\} = \frac{1}{s} \tanh \frac{s}{2}$$

13. Find the general solution of the system $\mathbf{X}' = \mathbf{A}\mathbf{X} + \mathbf{F}(t)$ where $\mathbf{A} = \begin{pmatrix} 1 & -3 \\ 2 & -4 \end{pmatrix}$ and $\mathbf{F}(t) = \begin{pmatrix} e^t \\ e^t \end{pmatrix}$.

14. Find the general solution of the system $\frac{dx}{dt} = 2x - y$, $\frac{dy}{dt} = 4x + 2y$ using the eigenvalue method.

15. Find the general solution of the system $\frac{dx}{dt} = -3y$ and $\frac{dy}{dt} = -x - y$ and sketch the corresponding phase portrait. Is the origin an attractor or repeller?

16. Solve the system

$$\mathbf{X}' = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \mathbf{X} + \begin{pmatrix} 2e^{3t} \\ e^{3t} - e^{-3t} \end{pmatrix}$$

using the variation of parameter method.

17. Solve the initial value problem

$$\mathbf{X}' = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix} \mathbf{X} + \begin{pmatrix} 0 \\ t \\ e^{4t} \end{pmatrix}$$

$$\text{with } \mathbf{X}(0) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}.$$

18. Find the general solution of the differential equation $x^2y'' + xy' - y = x$ for $x > 0$.

19. Find the first four non-zero terms in a power series solution about $x = 0$ for $(1 - x^2)y'' - 4xy' + 6y = 0$

20. Compute the exponential matrix e^{tA} for $A = \begin{pmatrix} -4 & 6 \\ -3 & 5 \end{pmatrix}$ using the Laplace transform method. Then find a fundamental matrix $\phi(t)$ for the differential equation $\mathbf{X}' = \mathbf{A}\mathbf{X}$ satisfying $\phi(0) = \mathbf{I}$.

21. Find the critical points and the phase-line portrait of the autonomous differential equation $y' = y^2(y^2 - 1)$. Classify the critical points as asymptotically stable, unstable or semi-stable. By hand sketch typical curves in the regions of the xy -plane determined by the graphs of the equilibrium solutions.