Solutions Assignment 13, MATH 333, Fall, 2009

Appendix II, 6: (a) $\mathbf{AB} = -16$, (b) $\mathbf{BA} = \begin{pmatrix} 78 & 54 & 99 \\ 104 & 72 & 132 \\ -26 & -18 & -33 \end{pmatrix}$,

(c) $\mathbf{BA})\mathbf{C} = \begin{pmatrix} 15 & -18 & 21 \\ 20 & -24 & 28 \\ -5 & 6 & -7 \end{pmatrix} \begin{pmatrix} 1 & 2 & 4 \\ 0 & 1 & -1 \\ 3 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 78 & 54 & 99 \\ 104 & 72 & 132 \\ -26 & -18 & -33 \end{pmatrix}$,
(d) $(\mathbf{AB})\mathbf{C}$ is not defined because $\mathbf{AB}$ is 1×1 and C is 3×3.

Appendix II, 8: (a) $\mathbf{A} + \mathbf{B}^T = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} + \begin{pmatrix} -2 & 5 \\ 3 & 7 \end{pmatrix} = \begin{pmatrix} -1 & 7 \\ 5 & 11 \end{pmatrix}$, (b) $2\mathbf{A}^T - \mathbf{B}^T = \begin{pmatrix} 2 & 4 \\ 4 & 8 \end{pmatrix} - \begin{pmatrix} -2 & 5 \\ 3 & 7 \end{pmatrix} = \begin{pmatrix} 4 & -1 \\ 1 & 1 \end{pmatrix}$, (c) $\mathbf{A}^T(\mathbf{A} - \mathbf{B}) = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} 3 & -1 \\ -3 & -3 \end{pmatrix} = \begin{pmatrix} -3 & -7 \\ -6 & -14 \end{pmatrix}$

Appendix II, 14: $\begin{pmatrix} -9t+3 \\ 13t-5 \\ -6t+4 \end{pmatrix} + \begin{pmatrix} -t \\ 1 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ 8 \\ -6 \end{pmatrix} = \begin{pmatrix} -10t+1 \\ 13t-12 \\ -6t+14 \end{pmatrix}$

Appendix II, 20: Since $det(\mathbf{A}) = 27$ the matrix $\mathbf{A}$ is nonsingular. The cofactors are $M_{11} = -1, M_{12} = 4, M_{13} = 22, M_{21} = 7, M_{22} = -1, M_{23} = -19, M_{31} = -1, M_{32} = 4, M_{33} = -5$, so

$$A^{-1} = \frac{1}{27} \begin{pmatrix} -1 & 4 & 22 \\ 7 & -1 & -19 \\ -1 & 4 & -5 \end{pmatrix}^T = \frac{1}{27} \begin{pmatrix} -1 & 7 & -1 \\ 4 & -1 & 4 \\ 22 & -19 & -5 \end{pmatrix}$$

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Appendix II, 24: Since $det(\mathbf{A}(t)) = 2e^{2t} \neq 0$, $\mathbf{A}(t)$ is nonsingular for all t and

$$\mathbf{A}(t)^{-1} = \frac{1}{2} e^{-t} \begin{pmatrix} \sin(t) & 2\cos(t) \\ -\cos(t) & 2\sin(t) \end{pmatrix}$$

Appendix II, 34: $\begin{pmatrix} 1 & 1 & -3 & | & 6 \\ 4 & 2 & -1 & | & 7 \\ 3 & 1 & 1 & | & 4 \end{pmatrix} \Longrightarrow \begin{pmatrix} 1 & 1 & -3 & | & 6 \\ 0 & -2 & 11 & | & -17 \\ 0 & -2 & 10 & | & -14 \end{pmatrix} \Longrightarrow \begin{pmatrix} 1 & 0 & \frac{5}{2} & | & -\frac{5}{2} \\ 0 & 1 & -\frac{11}{2} & | & \frac{17}{2} \\ 0 & 0 & -1 & | & 3 \end{pmatrix}$
$\Longrightarrow \begin{pmatrix} 1 & 0 & 0 & | & 5 \\ 0 & 1 & 0 & | & -8 \\ 0 & 0 & 1 & | & -3 \end{pmatrix}$

Appendix II, 50: The characteristic equation is $det(\mathbf{A} - \lambda\mathbf{I}) = \begin{vmatrix} 1-\lambda & 1 \\ \frac{1}{4} & 1-\lambda \end{vmatrix} = (\lambda - \frac{3}{2})(\lambda - \frac{1}{2}) = 0$ and has solutions $\lambda_1 = \frac{3}{2}$ and $\lambda_2 = \frac{1}{2}$. For $\lambda_1 = \frac{3}{2}$ the solution of

$$\begin{pmatrix} -\frac{1}{2} & 1 & | & 0 \\ \frac{1}{4} & -\frac{1}{2} & | & 0 \end{pmatrix} \Longrightarrow \begin{pmatrix} 1 & -2 & | & 0 \\ 0 & 0 & | & 0 \end{pmatrix}$$

is $k_1 = 2k_2$. With $k_2 = 1$ we get the eigenvector $\mathbf{K}_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$. Similarly for $\lambda_2 = \frac{1}{2}$ we get

$$\begin{pmatrix} \frac{1}{2} & 1 & | & 0 \\ \frac{1}{4} & \frac{1}{2} & | & 0 \end{pmatrix} \Longrightarrow \begin{pmatrix} 1 & 2 & | & 0 \\ 0 & 0 & | & 0 \end{pmatrix}$$

so $k_1 = -2k_2$ and with $k_2 = 1$ we get $\mathbf{K}_2 = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$.

Exercises 8.1, 6: With $\mathbf{X} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ we get

$$\mathbf{X}' = \begin{pmatrix} -3 & 4 & 0 \\ 5 & 0 & 9 \\ 0 & 1 & 6 \end{pmatrix} \mathbf{X} + \begin{pmatrix} e^{-t}\sin(2t) \\ 4e^{-t}\cos(2t) \\ -e^{-t} \end{pmatrix}$$

Exercise 8.1, 22: Since $\mathbf{X}_p' = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \end{pmatrix} + \begin{pmatrix} -5 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ it follows that

$$\mathbf{X}_p' = \begin{pmatrix} 2 & 1 \\ 1 & -1 \end{pmatrix} \mathbf{X}_p + \begin{pmatrix} -5 \\ 2 \end{pmatrix}$$

Additional Exercise: With $x_1 = y$ and $x_2 = y'$ the initial value problem $y'' - 7y + 10y = 12\cos(t)$, $y(0) = 1$, $y'(0) = 0$ is equivalent to the initial value problem

$$\begin{aligned} x_1' &= x_2 \\ x_2' &= -10x_1 + 7x_2 + 12\cos(t) \end{aligned}$$

with initial conditions

$$\begin{aligned} x_1(0) &= 1 \\ x_2(0) &= 0 \end{aligned}$$