Practice exam for Exam 3, MATH 333, Fall, 2009

1. Solve the initial value problem $y'' - 2y' + y = te^t \sin(t)$, $y(0) = y'(0) = 0$ using the Laplace transform method.

2. Solve the initial value problem $y'' + 2y' + y = f(t)$, $y(0) = y'(0) = 0$ with

$$f(t) = \begin{cases} e^t & \text{for } t \text{ in } [0,1) \\ 0 & \text{for } t \text{ in } [1,\infty) \end{cases}$$

using the Laplace transform method.

3. Define

$$f(t) = \begin{cases} 1-t & \text{for } 0 \le t < 2 \\ t-3 & \text{for } 2 \le t < 4 \end{cases}$$

and $f(t+4) = f(t)$. Find $\mathscr{L}\{f(t)\}$.

4. Find $\mathscr{L}^{-1}\{\frac{2s^2+s-4}{s^3-s^2-2s}\}$ and $\mathscr{L}^{-1}\{\frac{3s+4}{(s+1)(s+2)^2}\}$.

5. Find the Laplace transform of $e^{-t}(1+t^2)$ and $e^{-t}\sin^2(t)$.

6. Solve the equation

$$\int_0^t y(\tau)\sin(t-\tau)d\tau = t^2$$

7. Solve the equation

$$y(t) - \int_0^t e^{-\tau} y(t-\tau)d\tau = e^t$$

8. An inductor of 2 henrys is connected in series with a resistor of 12 ohms, a capacitor of 1/16 farads, and a battery assumed to be 300 volts. The initial instantaneous charge q on the capacitor is 10 coulombs, and the initial current is $q'(0) = 0$. Formulate an initial value problem that models the circuit described, and solve the model using the Laplace transform method.

9. Solve the system of linear equations using the Gauss or Gauss-Jordan method:

$$\begin{aligned} x_1 + x_2 + 2x_3 &= 1 \\ 2x_1 - x_2 + x_3 &= 2 \\ 4x_1 + x_2 + 5x_3 &= 4 \end{aligned}$$

10. Find the inverse matrix $\mathbf{A}^{-1}$ of the matrix $\mathbf{A} = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & -2 \\ 0 & 1 & 1 \end{pmatrix}$ using (i) Gauss-Jordan elimination, and (ii) the cofactor formula.

11. Find the eigenvalues and eigenvectors of the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{pmatrix}$$

12. For the initial value problem $(*) : y''' + ty'' - t^2y' + e^ty = \sin(t)$, $y(0) = y'(0) = 1$, $y''(0) = 0$ find a linear first order system of differential equations for functions $x_1(t), x_2(t), x_3(t)$ and a corresponding initial value problem $(**)$ such that $y(t) = x_1(t)$ will solve $(*)$ if and only if $(x_1(t), x_2(t), x_3(t))$ solve $(**)$.

13. Write the system of differential equations

$$\begin{aligned} x' &= 3x - 2y + 2 - 2e^t \\ y' &= x - e^t \\ z' &= -x + y + 3z + e^t - 1 \end{aligned}$$

in matrix form. Then show that

$$\mathbf{X} = \begin{pmatrix} e^t \\ 1 \\ 0 \end{pmatrix}$$

is a solution of the system.

14. Show that the general solution of the linear system $\mathbf{X}' = \mathbf{AX}$ with

$$\mathbf{A} = \begin{pmatrix} 3 & 1 & 1 \\ 2 & 2 & 1 \\ -4 & -2 & -1 \end{pmatrix}$$

is given by

$$\mathbf{X} = c_1 \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} e^t + c_2 \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} e^t + c_3 \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} e^{2t}$$

for c_1, c_2, c_3 arbitrary real constants.