Practice exam for Exam 2, MATH 333, Fall, 2009

1. Find the solution of the differential equation $y'' = -3y' + 10y - 3x(e^x + 1)$
2. Find the solution of the differential equation $y'' = 2y' + y + e^x \cos(x)$
3. A spring attached to the ceiling is stretched one foot by a four pound weight. The mass is set into motion in a medium that imparts a damping force numerically equal to the velocity. The mass is pulled down by 1 foot and released. Find the position $x(t)$ as a function of time t.
4. Solve the differential equations $x^2y'' - xy' + y = 0$ and $x^2y'' + 3xy' + 10y = 0$.
5. Find the general solution of the differential equation $y'' = xy' + 1$ (check your solution) and the particular solution with initial condition $y(0) = 0$, $y'(0) = -1$.
6. Check that $y_1 = x^{-1}$ and $y_2 = x^2$ are linearly independent solutions of the homogeneous differential equation corresponding to the differential equation $y'' = \frac{2}{x^2}y - \frac{1}{x^2} + 3$ on $(0, \infty)$. Find a particular solution of the inhomogeneous equation.
7. Solve the initial value problem
$$\frac{dx}{dt} = 4y + 3, \quad \frac{dy}{dt} = -x + 2, \quad x(0) = 0, y(0) = 1$$
8. Find the solution of the differential equation
$$x^2y'' - 2xy' + 2y = 2x^3$$
9. A model for a nonlinear mass-spring system is $\frac{d^2x}{dt^2} + x - x^3 = 0$, where x is the displacement and mass m. Solve this for $\frac{dx}{dt}$ by multiplying by $\frac{dx}{dt}$ and integrating.
10. Solve the eigenvalue problem $y'' + \lambda y = 0$, $y'(0) = 0$, $y'(\frac{\pi}{2}) = 0$.
11. Find the first three terms of each of the two linearly independent power series solutions of the differential equation
$$y'' - xy' + x^2y = 0$$