Assignment VII, Math 187

Dr. Holmes

July 6, 2004

This assignment is due Monday, July 12. There is likely to be another assignment distributed before this one is due.

1. Write out a complete proof that if $c|a$ and $c|b$, then $c|xa + yb$, where x and y are any integers. You may assume familiar algebraic properties of the integers.

2. Convert $54_{\texttt{ten}}$ and $31_{\texttt{ten}}$ to base 6.

   Construct addition and multiplication tables for base 6.

   Add and multiply the base six versions of $54_{\texttt{ten}}$ and $31_{\texttt{ten}}$, carrying out all calculations in base six.

   Convert the results back to base ten and check that you have the correct answers.

3. Compute $\texttt{gcd}(5898{,}1113)$ using the Euclidean algorithm.

   Find integers x and y such that $5898x + 1113y = 9$.

   Is it possible to find integers x and y such that $5898x + 1113y = 20$? If not, why not?

4. Let F_i be the ith Fibonacci number (as usual). Prove that $\texttt{gcd}(F_i, F_{i+1}) = 1$ for every i. Hint: use proof by induction.

5. Define a relation R on the positive integers by $x\,R\,y$ iff $\texttt{gcd}(x, y) > 1$. Is this relation symmetric? reflexive? transitive? Explain in each case why it does or does not have the property.