

Remarks for Test II Review

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I currently have a draft which has something like eight questions with a single essay question (with a number of choices). The draft probably doesn't look much like the final version of the test (the final test is likely to be shorter, for one thing). I encourage questions about this document on Tuesday; I also encourage you to keep looking at it because I will probably revise it as I keep thinking about the exam...

Read the history readings with an eye to the attitudes of mathematicians of different cultures and periods toward numbers of different kinds (fractions, irrationals, negative numbers, and complex numbers). Attitudes to whether these are legitimate numbers and willingness to use them in calculations should both be considered.

Read the historical material in general, not just for the specific point above; I might come up with something else to ask about.

Be familiar with Russell's constructions of the various number systems. If I ask you to compare the natural number 2, the positive rational number 2, and the positive real number 2 (for example) you should be able to tell me what each of these things is like.

I'll review Dedekind's definition of a reals as a "Dedekind cut" and compare it with Russell's definition.

Be generally familiar with the computations involved in the solution of the cubic equation. I will very likely ask for some related computation, but not for a complete solution of a cubic, as that probably would take too much time.

I might ask some kind of question about geometric solution of the quadratic equation – any geometric construction I ask about would actually be given in the question with diagrams.

I could ask about the Eudoxian theory of proportions for irrational line segments (my current draft does not have such a question); in this case the definitions from Euclid would be given.

I could ask a question about the implementation of the complex numbers as geometric transformations of the plane. A review of the trigonometric form of complex numbers wouldn't hurt. Be sure you do remember how to do simple calculations with complex numbers.

I'm not going to ask any kind of question about my implementation of infinitesimals as functions, but I might ask about the computation of derivatives using infinitesimals as it was done by 17th century mathematicians, and ask for an explanation of what is wrong with it logically.

Some kind of question on the implementation of notions of the geometry of Euclid or Hilbert using analytic geometry can be expected.

Possible essay questions:

Compare and contrast my positions 1,2,3 on constructions of numbers. Which position do you agree with and why (you might have different opinions with regard to different number systems).

Compare and contrast Russell's notion of "propositional function" with the more modern notion of set. Discuss notational details of Russell's notation and compare the modern approach in which a relation is a set of ordered pairs (so the notion of relation depends on the notion of pair) with Russell's implementation of the ordered pair as a particular relation. Is Russell's postulation of propositional functions and use of these to implement various kinds of numbers preferable to earlier mathematicians simply postulating the numbers they needed (how is he on the "theft vs. honest toil index).

Do you think that the natural number 2, the rational number 2, and the real number 2 ought to be the same abstract object or different abstract objects? Compare and contrast the functions of these different (or identical?) abstractions.

Does the definition of instantaneous velocity using limits answer Zeno's paradox of the arrow? I'll discuss in class the question as to what interval(s) of time the limit definition appeals to in determining the velocity.