

Math 333 Test 4

Dr. Holmes

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The exam begins at 7:40 am and ends at 8:35 am on each of the two days that it is given.

You may have your test paper, your calculator, a writing instrument, and drafting tools (if desired) on your desk. Nothing else is permitted.

Use of any calculator is permitted on this exam, but be sure to provide work when it is requested. In general, if you use your calculator show the setup for your calculations on your paper.

You are not permitted to use cell phones or palmtop computers as calculators, for security reasons.

1. Compute the inverse of the matrix

$$\begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix}$$

using row operations and showing all steps. You may use a calculator to carry out the row operations, but you must describe the row operations and show the matrices on your paper.

Use a matrix calculation with this inverse matrix to solve the system of equations

$$x + 3y = 5$$

$$2x + 5y = 4$$

You may use a calculator to carry out this computation, but you must set it up on your paper.

2. The matrices

$$\begin{pmatrix} 2 & 3 \\ 0 & -1 \\ -1 & 6 \end{pmatrix}$$

and

$$\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$$

can be multiplied in one order but not the other.

Write down the two matrices in the order in which they can be multiplied, and carry out the multiplication. Your work should show evidence that you know how to multiply matrices by hand.

Explain why they cannot be multiplied in the other order.

3. A system of tanks with various pipes transferring water with different percentages of HCl at different rates is shown. Set up a system of differential equations and initial value problem whose solution will describe the amount of hydrochloric acid in each tank at time t .

It is good for extra credit if you can solve the initial value problem and state the amount of HCl in each tank at any time t .

4. Solve the system of linear differential equations with constant coefficients using the method of eigenvectors and eigenvalues.

You must show hand calculations to find the eigenvectors and eigenvalues for full credit.

If the eigenvalues are complex, you need to give real solutions to the equations.

$$x' = 5x + 2y$$

$$y' = -x + 2y$$

Give your final answer in the form

$$x(t) = \dots$$

$$y(t) = \dots$$

5. The set of vectors

$$\begin{pmatrix} 1 \\ -6 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 21 \\ 1 \end{pmatrix}$$

is linearly dependent.

Verify that it is linearly dependent by computing a determinant (showing hand calculations).

Find a nontrivial linear combination of these vectors which is equal to the zero vector, by appropriate matrix methods.

6. The second order differential equation

$$x'' - 3x' + 2x = 0$$

has characteristic equation $\lambda^2 - 3\lambda + 2 = 0$ with roots 1 and 2, so the general solution of this equation is of the form $c_1e^t + c_2e^{2t}$.

Convert this equation to a system of two first-order equations in two unknowns, and show that this system has the same characteristic polynomial as the second-order equation from which it is derived (by computing the appropriate determinant explicitly).

You don't need to go farther than computing the eigenvalues of the matrix.

7. Verify that

$$\begin{pmatrix} e^{2t} \\ e^{2t} \end{pmatrix}$$

and

$$\begin{pmatrix} e^{3t} \\ \frac{1}{2}e^{3t} \end{pmatrix}$$

are solutions of

$$x' = 4x - 2y$$

$$y' = x + y$$

Verify that these are linearly independent solutions using the Wronskian (show hand calculations for the evaluation of the determinant).

Write the general solution to this system of equations in the format

$$x(t) = \dots$$

$$y(t) = \dots$$

Find the solution to this equation satisfying the initial condition

$$x(0) = 1; y(0) = 2$$

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8. Solve the system of linear differential equations with constant coefficients using the method of eigenvectors and eigenvalues.

You must show hand calculations to find the eigenvectors and eigenvalues for full credit.

If the eigenvalues are complex, you need to give real solutions to the equations.

$$x' = x - 2y$$

$$y' = 2x + y$$

Give your final answer in the form

$$x(t) = \dots$$

$$y(t) = \dots$$