Math 333 Test III

Dr. Holmes

April 3, 2003

This test lasts from 7:40 am to 8:35 am on each of the two days on which it is administered. The test conditions are the same as on past tests: you are allowed to have a writing instrument, your test paper, drafting tools if desired, and a calculator with no graphing or symbolic computation capabilities. Cell phones must be **turned off**. You may not use cell phones or palmtop computers as calculators.

1. Find a particular solution to the differential equation

$$y'' - y = e^t$$

using the method of variation of parameters. If you use some other method, you will receive very limited credit (I know you can do this by undetermined coefficients or Laplace transforms, but that is not what I am looking for).

The promised equations are

$$v_1' y_1 + v_2' y_2 = 0$$

$$v_1' y_1' + v_2' y_2' = g(t)$$

Also as promised, I provide no explanation: you are supposed to know what this means and how to use it to solve the problem.

2. Laplace transforms

(a) The function $y = \sin(2t)$ satisfies the conditions $y'' = -4y$, $y(0) = 0$, $y'(0) = 2$. Use these conditions, along with rules for calculation with Laplace transforms, to compute the Laplace transform of $\sin(2t)$ (I assume that you know what this transform is from a table, so just stating it without any calculation carries very little credit).

(b) Write down the Laplace transforms of the following functions. Show any work that you do (some of these you may just be able to write down from memorized tables).

i. $\cos(5t)$

ii. $e^{-t}\sin(t)$

iii. $t\cos(t)$

iv. $H(t-1)(t-1)^2$

3. Use the method of Laplace transforms to solve the initial value problem

$$y'' + 3y' + 2y = e^{-3t}; y(0) = 0, y'(0) = 1$$

Show all work. Solving the equation by any other method carries very little credit.

4. Give a numerical approximation to $y(1)$, where y is the solution of the initial value problem $y' = t + y^2, y(0) = 0$, using four iterations of Euler's method (so $\Delta t = .25$). Put your work in a tabular format so that I can follow it. Do all calculations to at least four decimal places. Be sure to clearly state at the end what your approximation to $y(1)$ actually is.

5. An inductor (1 H) and a capacitor (.25 F) are connected in series and an electromotive force of $\sin(t)$ volts. There is no current nor any charge on the capacitor when $t = 0$. Determine the charge on the capacitor at time t and the current at time t. You may use any technique for solving differential equations that you prefer.

6. Determine the inverse Laplace transforms of the following functions.

(a)

$$\frac{1}{s^2+9}$$

(b)

$$\frac{e^{-s}}{s+1}$$

(this involves a Heaviside function)

(c)

$$\frac{s}{s^2-5s+6}$$

(use partial fractions)

7. Solve the initial value problem

$$y' - y = f(t),$$

where $f(t)$ is defined as t for $t < 2$ and 2 for $t > 2$ (see graph of $f(t)$ provided), using the method of Laplace transforms. Give your final answer in a form which does not involve Heaviside functions (i.e., in piecewise form). (You are not *required* to use Laplace transforms; if you actually know how to solve the problem without transforms and can do it correctly, this will be acceptable, though I think it is probably harder)

8. Give a numerical approximation to $y(1)$, where y is the solution of the initial value problem $y' = t^2 + y, y(0) = 0$, using two iterations of the second-order Runge-Kutta method (so $\Delta t = .5$). Put your work in a tabular format so that I can follow it. Do all calculations to at least four decimal places. Be sure to clearly state at the end what your approximation to $y(1)$ actually is.