# Math 333 Test II, Summer '09

Dr. Holmes

June 30, 2010

The exam will begin at 11:50 am and end at 1:30 pm. You may use a writing instrument and a plain scientific calculator without graphing or symbolic computation capability. Write your name on the exam paper and on the blue book(s) you use, and return both at the end of the exam. Be sure to record the ID number on your blue book which will be used to post your grade on my web page. Write all work in your blue book(s): work written on the test paper will not be noticed. Cell phones must be turned off and out of sight.

1. Sketch the direction field for the differential equation

$$y' = -(y+1)(y-3).$$

Sketch in the equilibrium solutions. Identify each equilibrium solution as stable or unstable. Sketch in representative solutions of each kind. There is no need to solve the equation: these are purely qualitative sketches.

2. A circuit has a resistor (R=2) and a coil (L=1) in it. At time 0, there is no current in the circuit. From time t=0 to 3, a constant voltage of 20 is supplied; after time 3 no voltage is supplied. Set up appropriate differential equations and initial value problems to find formulas for the current in the circuit at time $t$ (a formula for $0 \le t \le 3$ and a formula for $t \ge 3$).

   The form of the differential equation here is $LI' + RI = E(t)$.

3. Find the general solution for each of the following equations.

   (a)
   $$y'' + 3y' + 2y = 0$$

   (b)
   $$y'' - 6y' + 9y = 0$$

   (c)
   $$y'' + 2y' + 5y = 0$$

4. Find the solution to the initial value problem

$$y'' + 3y' + 2y = 0; y(0) = 3; y'(0) = -4.$$

5. Find the general solution of the equation

$$y'' + 3y' + 2y = \sin(2t).$$

6. A mass of 2 kg is suspended at the end of a spring with constant 18. The spring is allowed to come to equilibrium: at time 0 the spring is set into motion from equilibrium position at velocity 0.5 m/sec. Neglecting

friction, determine the position of the weight at time $t$. State the amplitude and frequency of the oscillation of the spring.

What coefficient of friction $\mu$ would need to be introduced for this system to become critically damped?

The general form of the equation here is $mx'' + \mu x' + kx = 0$.