The exam begins at 1240 am and ends at 135 pm. You are allowed your writing instrument, your test paper, and your non-graphing calculator. Good luck!
1. Solve the homogeneous second-order linear differential equations and initial value problems.

(a) $y'' + 6y' + 9y = 0$ Find the general solution.

(b) $y'' + 2y' + 5y = 0$ Find the general solution.

(c) $y'' + 5y' + 4y = 0; y(0) = 1; y'(0) = 2$. Find the general solution to the differential equation, then solve the initial value problem.
2. Solve the inhomogeneous second order linear differential equation by the method of undetermined coefficients.

Solve the equation \( y'' + 3y' + 2y = \sin(3t) \). Give a particular solution found by undetermined coefficients (the method of section 4.5) and the general solution. You are permitted to use the complex method if you know how to do it. Solving the problem by Laplace transforms does not carry full credit.
3. An undamped harmonic oscillator

A mass of one kilogram is suspended at the end of a spring with the spring constant \( k = 9 \). The spring is stretched 20 cm and released with a sharp tap which propels it upward at 10 cm/sec. Set up and solve a differential equation describing the resulting motion. Determine the frequency, amplitude and phase of the motion.
4. An electric circuit

A circuit contains a coil with inductance (coiliness?) 1, a resistor with resistance 3, and a capacitor with capacitance 0.5. An input voltage of \( \sin(t) \) is supplied. The current in the circuit is initially zero and the charge on the capacitor is initially three units. Write expressions for the charge on the capacitor at time \( t \) and the current in the circuit at time \( t \). Break both of these into transient parts (terms that go to zero) and steady state parts (those which do not go to zero).

You may use any mathematical technique you prefer to solve this problem. I supply some information in symbolic form.

\[
LQ'' + RQ' + \frac{1}{C}Q = E(t)
\]

\[
Q' = I
\]

This is just a reminder; you should know what all these letters mean.
5. A differential equation with a piecewise defined forcing term.

Write the function \( f(t) = 1 - H(t - 1) \) as a piecewise defined function. Sketch its graph.

Solve the initial value problem

\[
y' + 4y = 1 - H(t - 1); \quad y(0) = 0
\]

Notice that this is a first-order equation, not a second-order equation (no second derivatives; no sines or cosines); this makes it easier!

I suggest using Laplace transforms, though if you can solve it without using Laplace transforms you are welcome to do so.

Your final answer should not involve Heaviside functions (so if you use LTs you should have a final step where you convert an answer involving Heaviside functions to piecewise form).
6. Do one of the two parts. If you do both, your best work will count. If you do well on both, you may get some extra credit.

(a) You are given the fact (which you already know) that one of the solutions of

\[ y'' + 2y' + y = 0 \]

is \( y_1(t) = e^{-t} \). Find the general solution to this equation by the method of section 4.1, that is, by writing the general solution \( y \) in the form \( y = uy_1 \), plugging this into the differential equation and solving for the unknown function \( u \). Remember the product rule!
(b) The differential equation

\[ 2y \, dx + x \, dy = 0 \]

is not exact. We tell you for free that it has an integrating factor depending only on \( x \), that is there is a function \( \mu(x) \) such that

\[ 2y\mu(x) \, dx + x\mu(x) \, dy = 0 \]

is an exact equation. Find \( \mu(x) \) and solve the differential equation.