This homework is due Friday, February 4th, at the beginning of class.

1. Simplify the following negative statement as far as possible using de Morgan’s laws and double negation. Show step by step work.

   \[ \neg(A \land \neg(B \lor \neg C)) \]

2. The English construction “Neither P nor Q” is for purposes of this exercise only represented as \( P \downarrow Q \) and defined by the truth table

   \[
   \begin{array}{c|c|c}
   P & Q & P \downarrow Q \\
   \hline
   T & T & F \\
   T & F & F \\
   F & T & F \\
   F & F & T \\
   \end{array}
   \]

   Write an expression equivalent to \( P \downarrow Q \) using only negation and disjunction.

   Write an expression equivalent to \( P \downarrow Q \) using only negation and conjunction.

   Write an expression equivalent to \( \neg P \) using only neither/nor.

   Write expressions equivalent to \( P \lor Q, P \land Q, P \rightarrow Q, P \leftrightarrow Q \) using only neither/nor.

   Every expression (in however many letters) defined by a truth table can be written in terms of negation, conjunction and disjunction, so in fact they can all be written in terms of neither/nor. This is amusing, though in most contexts not terribly useful.
3. In the following problems, do not use substitutions of equivalent expressions (such as deMorgan’s laws in the style used in the first problem). Give a formal proof of \((P \rightarrow Q) \land (Q \rightarrow R) \rightarrow (P \rightarrow R)\) in the same style I use in the first example in the notes.

4. Write formal proofs of both of the deMorgan laws in the style I use in the notes.
   \[
   \neg(A \land B) \leftrightarrow (\neg A \lor \neg B) \\
   \neg(A \lor B) \leftrightarrow (\neg A \land \neg B)
   \]
   I’m likely to do one of these in class, which will make it a bit easier.

5. The converse of the theorem that I give in the notes to verify that the technique of Proof by Cases is also a theorem: prove \(((A \lor B) \rightarrow C) \rightarrow ((A \rightarrow C) \land (B \rightarrow C))\) in the style I use in the notes.

6. Prove \(\neg(P \rightarrow Q) \leftrightarrow P \land \neg Q\). This is an exercise in careful proofs of negations and proofs by contradiction.