Some of these questions may literally be on the exam. Don’t discount this possibility. The questions in the exam fall into various categories, and the instructions will ask you to do the easiest problems in each category before the others (so that there is a baseline performance from which I can work: I do not know how much of the exam the typical student will be able to write).

It is not impossible that the exam will be exactly a subset of this practice exam, though likely some numbers will be changed to protect the innocent. It will consist of problems in this spirit. It will be shorter. It will probably be long enough that I will not expect a student to write the entire paper, but the instructions will require you to write at least one problem of each kind, then as many others as you can.

There will be propositional logic questions, formal arithmetic questions, general algebra questions, and limit proofs.

1 **Logic Questions**

1. If I want to put a really easy question on I might put some kind of truth table question, but I am not sure length will permit.

2. Give a proof of \(((P \rightarrow Q) \land (Q \rightarrow R)) \rightarrow (P \rightarrow R)\) in the style taught in class.

3. Give a proof of \((P \rightarrow Q) \leftrightarrow \neg(P \land \neg Q)\) in the style taught in class. Remember that you are not allowed to use truth table reasoning or apply de Morgan’s laws.

4. State and prove one of de Morgan’s laws in the style taught in class.
2 Formal arithmetic questions

1. The axioms will be given.

2. Prove $S(x) + y = S(x + y)$ using the axioms alone.

3. Prove $1x = x$ using the axioms alone (well, you may also use the definition of $1$ and the theorem $s(x) = x + 1$).

4. Prove the right distributive law of multiplication using the axioms alone. (I might ask for any of the exercises proved in homework other than the commutative laws, which are a bit more involved – you will be given the theorems you proved earlier as tools to use).

3 General algebra questions

1. Prove the lemma “If $x + y = x$ then $y = 0$” from Spivak’s axioms (P1)-(P9) (which will be given). Then prove $a0 = 0$.

2. Given Spivak’s axioms and the theorem $a0 = 0$, first prove the lemma “if $x + y = 0$ then $y = -x$”, then prove that $a(-b) = -ab$.

3. Prove using either Spivak’s order axioms (P10)-(P12) and algebra skills, or his alternative set of axioms and algebra skills, that if $0 < a < b$ it follows that $a^2 < b^2$ (yes, you know the definition of $a^2$ as $aa$).

4. Prove that $|a| - |b| \leq |a - b|$ using the triangle inequality and algebra. Why does it follow immediately that $||a| - |b|| \leq |a - b|$?

5. Prove using familiar properties of even and odd numbers and algebra skills that the square root of 2 is not a rational number.

4 Limit proofs

1. Prove that $\lim_{x \to 3} 2x + 1 = 7$ in complete detail.

2. State and prove the limit law for addition (or subtraction, or constant multiples). [Multiplication is too complicated].