Math 311 Test I, Spring 2012

Dr. Holmes

February 15, 2012

The test will begin at 8:15 and end at 9:35. You may use your writing instrument and your test paper. If you require scratch paper, ask the instructor. Your grade on this test will be posted on the class web page by the serial number on the first inside page of your test paper.
1. Logic

(a) For any implication $A \rightarrow B$, explain what its converse and contrapositive are. Present a truth table showing that $A \rightarrow B$ is equivalent to its contrapositive and not equivalent to its converse.

(b) Negate each of the following sentences, using de Morgan's laws or the rules for negating quantifiers appropriately. Your answer can include logical symbols but does not have to do so.

i. All men are mortal.

ii. Angle $\alpha$ is acute or reflex.

iii. Any line has at least two points on it.
2. Four interpretations of incidence geometry are presented. One of them is a model. Identify it. For each of the other interpretations, state which of the three axioms of incidence geometry it satisfies and which it does not satisfy. When you state that a model does not satisfy an axiom, explain: your explanation should mention specific points and lines where appropriate.

(a) A point is one of six commuters, Sam, George, Fred, Barney, Betty and Wilma. A line is a carpool; a point (person) lies on a line (carpool) if the person belongs to the carpool. Sam and George commute together. Fred and Barney commute together. Betty and Wilma commute together.

(b) A point is one of the nine points in the diagram; a line is one of the curves shown (not all of which are straight lines).
(c) A point is one of the four numbers 1, 2, 3, 4. A line is one of the three element sets \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}. A point (number) lies on a line (set) if the number belongs to the set.

(d) A point is one of the three points shown; a line is one of the six lines shown. "lies on" has its usual meaning, but restricted to the points and lines shown.
3. Present three interpretations of incidence geometry which are not models, one in which axiom 1 does not hold but axioms 2 and 3 do hold, one in which axiom 2 does not hold but axioms 1 and 3 do hold, and one in which axiom 3 does not hold but axioms 1 and 2 do hold. You may use parts of the previous problem (at your own risk, your statements about them in the previous problem need to be correct). You need to provide explanations.
4. A model of incidence geometry is pictured in which none of the three Parallel Postulates which we have considered in connection with incidence geometry holds. Explain why each of the Parallel Postulates does not hold, using specific points and lines in your explanations (talk about specific lines and specific points not on those lines as counterexamples, and explain why they are counterexamples).
5. Two models of incidence geometry are presented. They are isomorphic, that is, they are essentially the same in a sense we have discussed in class: present a correspondence between the points and lines of interpretation 1 and interpretation 2 which witnesses this fact.
6. $A, B, C$ are three non collinear points. $D$ is a third point on line $AB$. $E$ is a third point on line $AC$.

Explain why $D$ and $E$ cannot be the same point, using the incidence axioms. You might want to reason by contradiction.

A diagram is provided, but you should not use facts about the picture in your reasoning.
7. Prove using the incidence axioms that there must be at least three distinct lines. You need to identify each place that you use each axiom.
8. Prove using the incidence axioms that for any point there is a line on which it does not lie. You need to identify each place that you use each axiom.