1. Determine the linearization of \( f(x, y) = \frac{x}{y} \) near \((2, 1, 2)\) (a function).

\[ \frac{\partial f}{\partial x} = \frac{1}{y} \quad \frac{\partial f}{\partial y} = -\frac{x}{y^2} \]

At \((2, 1, 2)\):
\[ (x, y) = 2 + 1(x-2) - 2(y-1) \]
\[ 1 \quad -\frac{2}{1^2} \]

Write an equation of the tangent plane to \( z = \frac{x}{y} \) at \((2, 1, 2)\).

\[ z = 2 + 1(x-2) - 2(y-1) \]
\[ z = x - y + 2 \]

\[ x = x - 2y + 2 \]

Estimate \( \frac{\log 1.08}{1.03} \) using the linearization.

\[ \frac{\log 1.08}{1.03} \sim \frac{2}{1} + 1(-0.02) - 2(0.03) \]
2. 14.5 Compute the directional derivative of \( f(x, y) = x^2 y \) in the direction of \( \langle 1, 1 \rangle \) at the point \( (2, 3) \):

\[
\nabla f(2, 3) \cdot \frac{\langle 1, 1 \rangle}{\sqrt{2}} = \begin{vmatrix}
2 & 1 \\
1 & 1 \\
\end{vmatrix} = \frac{16}{\sqrt{2}}
\]

Give a vector pointing in the direction in which this function is increasing fastest at \( (2, 3) \). What is this fastest rate of increase?

\[
\langle 12, 4 \rangle \text{ points in direction of fastest increase}
\]

\[
\text{and is norm } \sqrt{12^2 + 4^2}
\]

\[
\text{is the rate of increase in this direction.}
\]

\[
\langle 12, 4 \rangle \cdot \left( \frac{12}{\sqrt{12^2 + 4^2}}, \frac{4}{\sqrt{12^2 + 4^2}} \right) = \frac{12^2 + 4^2}{\sqrt{12^2 + 4^2}} = \sqrt{12^2 + 4^2}
\]
3. 14.6 Determine the partial derivatives of \( f(x, y) = x + y \) with respect to the polar coordinates \( r \) and \( \theta \), using the usual relations \( x = r \cos(\theta) \), \( y = r \sin(\theta) \) and the Chain Rule.

\[
\frac{df}{dr} = \frac{df}{dx} \cdot \frac{dx}{dr} + \frac{df}{dy} \cdot \frac{dy}{dr} = 2 \cos(\theta) - 1 \cdot r \sin(\theta)
\]

\[
\frac{df}{d\theta} = 2 \cdot \frac{dx}{d\theta} = -2rsin(\theta) - 1 \cdot r \cos(\theta)
\]
4. 14.7 Find all critical points of

\[ f(x, y) = 4x^3 + y^3 - 12x - 3y. \]

There are four of them. Classify each of the critical points as a local maximum, local minimum, or saddle point, using the second derivative test.

\[ \frac{df}{dx} = 12x^2 - 12 = 0 \quad x^2 = 1 \quad x = \pm 1 \]

\[ \frac{df}{dy} = 3y^2 - 3 = 0 \quad y^2 = 1 \quad y = \pm 1 \]

\[ D = 24x6y - 0 \]

at \((1, 1)\) \(D > 0\) \(24x > 0\) local max at\((1, 1)\)

at \((-1, 1), (-1, -1)\) \(D < 0\) saddle point

at \((-1, -1)\) \(D > 0\), \(24y < 0\) local max at \((-1, -1)\)
6. 15.1 Compute the integral of \( f(x,y) = x + 2y \) over the pictured region by setting up and evaluating an iterated integral. Set it up and evaluate it in both possible ways.

\[
\int_0^3 \int_1^2 (x + 2y) \, dy \, dx = \int_0^3 (x + 3) \, dy = \left[ \frac{x^2}{2} + 3y \right]_0^3 = \frac{9}{2} + 9 = \frac{27}{2}
\]

\[
\int_1^2 \int_0^3 (x + 2y) \, dx \, dy = \left[ \frac{x^2}{2} + 2xy \right]_1^2 = \left[ \frac{4}{2} + 4y \right] - \left( \frac{1}{2} + 2y \right) = \frac{5}{2} + 9 = \frac{27}{2}
\]
5. 14.8 Use the method of Lagrange multipliers to find the values of the coordinates \( x \) and \( y \) on the circle \( x^2 + y^2 = 1 \) which will maximize the area of the pictured square. You have the advantage that you ought to know the answer intuitively! You do need to use the method of Lagrange multipliers (section 14.8) to get credit.

\[
\text{maximize} \quad A = 4xy \\
\text{subject to} \quad x^2 + y^2 = 1
\]

\[
4y^2 + 4x^2 = \lambda (2x^2 + 2y^2)
\]

\[
2x = xy \quad \lambda = \frac{x}{y} = \frac{y}{x}
\]

\[
2y = xy \quad \text{so} \quad 2x^2 = 2y^2 \\
\text{so} \quad x^2 = y^2 \\
x = \pm y
\]

so \( x = \pm \frac{\sqrt{2}}{2} \)

\[
y = \frac{\sqrt{2}}{2}
\]

but only + val, make sure

choose \( x = \frac{\sqrt{2}}{2} \), \( y = \frac{\sqrt{2}}{2} \) (the angle is a square (co, as expected))