Math 275 Test II, Summer 2013

July 23, 2013
1. 13.1 Give a parameterization of the circle with center \((1, -2, 3)\) and radius 3 which is parallel to the \(xz\) plane.
2. 13.2 The velocity vector of a particle moving in the plane is given at each time $t$ by $\langle 2t, t^2 \rangle$, in both parts of this problem. Do both parts.

(a) What is its speed at time $t = 1$? What is its acceleration vector at time $t$? What is its acceleration vector at time $t = 1$?
(b) Suppose the position of the same particle at time \( t = 0 \) is given as \((1, -2)\). Give the formula for its position at any time \( t \). What is its position when \( t = 2 \)?
3. Do one of the two problems. If you do both parts, your best work will count. If you do well on both parts, you might get some extra credit.

(a) Determine an arc length parameterization of the circle parameterized by

\[ \mathbf{r}(t) = (2, 3 \cos(2t) + 4, 3 \sin(2t) - 1) \]

Hint: if you understand thoroughly what is being asked and what this circle is, this can be answered immediately without calculations, though it would be useful to give a few words of explanation in English. It can also be answered by doing calculations which in this case are straightforward.
(b) Determine the length of the portion of the helix \( \mathbf{r}(t) = (\cos(t), \sin(t), t) \) from \( t = 0 \) to \( t = \pi \). You are required to set up the integral (making it clear that you know how to do this in general) and evaluate it, which turns out to be very straightforward (use trig identities to simplify and you will see that the integral can be evaluated).
4. 13.4 Do one of the two parts. If you do both parts, your best work will count. If you do well on both parts, you might get some extra credit.

(a) Determine the curvature of the graph of $y = x^2$ at the point $(1, 1)$, the unit normal vector, and the radius of curvature. For a wee bit of extra credit, give the parameterization of the osculating circle. Hint: The graph is parameterized by $\langle t, t^2, 0 \rangle$. The unit normal is much easier to find than usual because you know the plane that it lies in.
(b) Determine the curvature of the curve parameterized by \( \mathbf{r}(t) = \langle t, t^2, t^3 \rangle \) at the point \( (2, 4, 8) \).
5. 13.5 Do one of the two parts.

(a) Explain the derivation of the formulas for tangential and normal components of acceleration, obtained by differentiating both sides of the formula $\mathbf{v}(t) = T(t)v(t)$. Your explanation will be graded on the use of English as well as math symbols: it would be especially useful to comment on what the various symbols mean. Some formulas are given on the last sheet of the exam, without supporting English – this you should supply to make it clear what everything means in your explanation.
(b) Compute the tangential and normal components of acceleration and the associated vectors at the moment $t = 1$ for the particle with position $(t, t^2, t^3)$ at each time $t$. 
6. Sketch contour lines $c = 1, c = 4, c = 9$ for the contour map of the function $z = x^2 + y^2$ (by convention, $c$ represents a fixed value of $z$ here). The same curves are contour lines for the contour map of the function $z = \sqrt{x^2 + y^2}$. Relabel them appropriately for the graph of this function.

What kinds of curves are the vertical traces (obtained by holding $x$ or $y$ at fixed values) of the function $z = x^2 + y^2$?
7. 14.2 Explain why \( f(x, y) = \frac{xy}{x^2+y^2} \) does not have a limit at the origin.

Hint: compute the limiting value of this function as \((x, y)\) approaches the origin along the \(x\)-axis.

Then compute the limiting value of this function as \((x, y)\) approaches the origin along the line \(y = x\).

Both of these calculations are representable as limits of functions of a single variable in a way you learned in your first semester. You do need to say something (very briefly) about the two values to draw your final conclusion.
8. 14.3 Compute the two partial derivatives and the four second partial derivatives of the function \( f(x, y) = x^2 + xy^3 + y^4 \). Verify that the mixed partials are equal.
1 Selected Formulas

Provided without comment, and without warranty express or implied. You are supposed to know what these mean; I will not offer explanations during the exam.

\[ \kappa(t) = \frac{|r'(t) \times r''(t)|}{|r'(t)|^3} \]
\[ \kappa(t) = \frac{|T'(t)|}{v(t)} \]
\[ \kappa(x) = \frac{f''(x)}{(1 + f'(x)^2)^{\frac{3}{2}}} \]
\[ a_T T = \frac{a \cdot v}{v \cdot v} \]