Math 275 Test I, Summer 2013

Dr. Holmes

July 23, 2013

The test will begin at 730 am and end at 910 am. What will actually happen at 910 am is that I will give a five minute warning. You may use your test paper, a non-graphing scientific calculator, and drafting tools if you like. Cell phones must be turned off and inaccessible to you.
1. Vectors $\mathbf{a}$ and $\mathbf{b}$ are pictured. Draw the vectors $\mathbf{a} + \mathbf{b}$, $\mathbf{a} - \mathbf{b}$, $-2\mathbf{a}$ and $2\mathbf{a} - 3\mathbf{b}$ on your paper. You are allowed to make additional copies of $\mathbf{a}$ and $\mathbf{b}$, which should be labelled and plausibly of the same length and direction.

Would the vector $\mathbf{a} \times \mathbf{b}$ point up into the air or down into your desk or table (assuming your test paper is sitting on your desk or table)?
2. We specify points $A = (1, 1), B = (3, 4)$ and $C = (2, -1)$.

Write the vectors $\overrightarrow{AB}, \overrightarrow{AC}$, and $\overrightarrow{BC}$ in vector notation.

Use vector calculations to determine the lengths of the three sides of the triangle and the magnitudes in degrees of each of the three angles.

It is useful to know that to compute $\angle ABC$ (for example) you want to use vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$; if you used $\overrightarrow{CA}$ instead you would get the supplement of the angle you wanted.
3. Add to my picture of copies of vectors \( \mathbf{a} \) and \( \mathbf{b} \) a picture of \( \text{proj}_b \mathbf{a} \), the projection of \( \mathbf{a} \) onto \( \mathbf{b} \). Your picture should clearly indicate the important geometric relation between these vectors.

Determine the decomposition of the vector \( \langle 1, 9, 3 \rangle \) into a component parallel to \( \langle 1, 2, 3 \rangle \) and a component perpendicular to \( \langle 1, 2, 3 \rangle \).
4. The corners of a parallelogram are $A = (1, 1, 1), B = (1, -2, 3), C = (3, 4, 5),$ and $D = (3, 1, 7)$. The relationships between points in the parallelogram are as sketched (not to scale!).

Determine the area of the parallelogram by computing a cross product of two vectors.

Determine the equation of the plane in which the four points lie in the form

$$ax + by + cz = d.$$
5. The lines parameterized by \( \mathbf{r}(t) = \langle 1, 1, 1 \rangle + t \langle 1, 0, -2 \rangle \) and \( \mathbf{s}(t) = \langle 2, 1, -1 \rangle + t \langle 0, 1, 1 \rangle \) intersect.

Determine their point of intersection.

Determine an equation for the plane which contains both lines.
6. Give vector parametric equations for the line of intersection between
the planes

\[ x - y + z = 2 \]

and

\[ x + 2y + 2z = 6. \]
7. Give cylindrical and spherical coordinates for the point $(1, 1, 1)$. A page of text containing the right formulas is attached.

Convert the equation $z = x + y + 1$ into the form $\rho = f(\theta, \phi)$: that is, express $x, y, z$ in spherical coordinates then solve for $\rho$ in terms of the other variables.
8. An object weighing 40 pounds is suspended from cables having indicated angles with the ceiling above (read from the picture). What are the force vectors and the magnitudes of the forces acting along the two cables? You may assume that the object is stationary (so gravity precisely cancels the forces on the cables) and the cables are not breaking.