A Step by Step Proof

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Theorem:

$$(a+b)^2 = a^2 + 2ab + b^2$$

Comments: Before this theorem can even be understood in the context of sections 4.1-2, it is necessary to notice that some definitions are needed. It's harder to notice this because the material is so familiar! Usually we will keep things somewhat more informal!

Definition: $2 = 1{+}1$

Definition: For any natural number n, define n^2 as nn.

Definition: $a + b + c$ will be defined as $a + (b + c)$; abc will be defined as $a(bc)$.

Proof:

$$(a+b)^2 = (a+b)(a+b)$$

by definition of n^2;

$$= (a+b)a + (a+b)b$$

by distributivity of multiplication over addition;

$$= a(a+b) + b(a+b)$$

by commutativity of multiplication (twice);

$$= (aa + ab) + (ba + bb)$$

by distributivity of multiplication over addition (twice);

$$= aa + (ab + (ba + bb))$$

by associativity of addition;

$$= aa + ((ab + ba) + bb)$$

by associativity of addition;

$$= aa + ((ab + ab) + bb)$$

by commutativity of multiplication;

$$= aa + ((1(ab) + 1(ab)) + bb)$$

by the identity property of multiplication;

$$= aa + ((1 + 1)(ab) + bb)$$

by distributivity of multiplication over addition;

$$= aa + (2(ab) + bb)$$

by definition of 2;

$$= a^2 + (2(ab) + b^2$$

by definition of n^2;

$$= a^2 + 2ab + b^2$$

by definitions of $a + b + c$ and abc

Notice that we are being *extremely* careful here; also notice that being extremely careful takes up time and space. We will not normally nit-pick to this extent!!!