

Counting Problems

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March 11, 2013

A selection of counting problems of various kinds.

1. License plates

License plates in a certain state have four letters and four digits on them. The four letters appear first.

- (a) If the above facts are all we know, how many license plates are possible?
- (b) If no letter can be repeated on a plate, and no digit can be immediately followed by the same digit, how many are possible?
- (c) How many plates have no A's?
- (d) How many plates have exactly two 6's on them?
- (e) How many plates would there be if the four letters and the four numbers could appear mixed up, in any order?
- (f) If no letter can be repeated on the plate and the letters must appear in alphabetical order, how many plates are there?

2. Rearranging words. State the number of different ways the letters in each word can be arranged.

(a) REARRANGE

(b) MISSISSIPPI

(c) SUBBOOKKEEPER

3. Arranging beads on a chain. You have a large supply of red blue and green beads. In how many ways can you make. . .

(Hint: in the closed necklaces you have to think about repeating patterns when you think about how many open chains will make a particular necklace)

- (a) an open chain of 11 beads
- (b) an open chain of 12 beads
- (c) a closed necklace of 11 beads (not too bad, though there is a special case you have to think about)
- (d) a closed necklace of 12 beads (quite hard)

4. Teams. 12 children are being divided into two teams to play a game.
- (a) In how many ways can you divide 12 children, 6 boys and 6 girls, into two teams to play a game, if the teams are called the Lions and the Tigers?
 - (b) In how many ways can you just divide the children into two teams?
 - (c) In how many ways can you divide the children into two teams if each team has to have three boys and three girls?

5. A florist has an unlimited supply of red roses, pink roses, yellow roses and white roses.
- (a) How many ways can you order a bouquet of a dozen roses for your sweetie? (all that matters is how many roses are in the bouquet).
 - (b) How many ways can you order a bouquet of a dozen roses if you want to have at least one of each kind?
 - (c) How many ways can you order a bouquet of a dozen roses if the store only has six pink roses?

6. Committee work. There are 24 people in the Math and Computer Science department. 18 of them are mathematicians and 6 are computer scientists.
- (a) In how many ways can a committee with six members be chosen with a designated chair and secretary?
 - (b) In how many ways can a committee with six members be chosen if the chair must be a mathematician and the secretary a computer scientist, or vice versa?
 - (c) In how many ways can the committee be chosen if four of its members must be mathematicians and two must be computer scientists. with the chair in one group and the secretary in the other?

7. A die (six sides, each with a different number on it) is to be painted using five colors.
- (a) How many ways are there to do this?
 - (b) How many ways are there to paint the cube so that at least two opposite sides are not the same color?
 - (c) How many ways are there to paint the cube so that no pair of opposite sides are the same color?

8. A telephone number in a certain country has 8 nonzero digits.
- (a) How many telephone numbers are there?
 - (b) How many telephone numbers have exactly four ones in them?
 - (c) How many telephone numbers have just as many odd digits as even digits?
 - (d) What is the probability of a randomly chosen telephone number having just as many odd digits as even digits?

9. What proportion of subsets of the digits 0-9 contain just as many odd numbers as even numbers? You need to determine how many subsets there are. Then you need to recall that you only need to consider subsets of each even size.

10. 26 students are in the junior class at a small private high school. All of them are taking at least one of English, French and Math. 12 are taking English. 13 are taking Math. 14 are taking French. 5 are taking English and Math. 4 are taking English and French. 6 are taking Math and French.

How many are taking all three courses?

How many are taking just English?

11. How many numbers from 1 to 1100 are not divisible by any of 2, 5, and 11? Hint: use inclusion/exclusion to determine how many are divisible by at least one of them.

12. Five college clubs each have 100 members. Each pair of the clubs contains exactly 10 members. Each intersection of three clubs contains 3 members. Strict college rules prevent anyone from belonging to 4 clubs at once. How many people are in the five clubs? Is this situation actually possible? [I actually don't know, I haven't done the full calculations; try analyzing it!]