Counting Problems

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April 26, 1998

These problems are intended for use in studying counting principles. Solutions to these problems will be distributed separately.

1. (thanks to Dr. Grantham) Suppose you're trying to guess my password, and that you know I've been foolish enough to construct it using exactly the following 13 characters, once each: s,b,g (my initials, in lower case); 1,9,5,7 (the year I was born); and C,H,E,R,Y,L (my wife's name, in upper case).

   (a) If you knew nothing about the order in which I had used those characters, how many different possibilities would there be for my password?

   (b) If you knew that I had kept the groups in the order listed (first my initials, then birth year, then wife's name), but may have scrambled the order *within* each group, then how many different possibilities would there be? [For example, bgs7915HLYREC would be one such possibility, but 5719gsbLRYCHE would not be, since the numbers precede the initials, and g57RECb91sHLY would not be, since the groups have been intertwined.]

(c) If you knew instead that I had kept the characters *within* each group in the relative order listed above, but that I might have "intertwined" the groups, then how many different possibilities would there be? [For example, CH1s9ERbg5Y7L would be one such possibility, but CH1s5ERbg9Y7L would not be, since the order of the 9 and 5 has been reversed.]

2. The reading list for a certain literature course consists of 7 novels by Ernest Hemingway and 9 by John Steinbeck. Each student must choose a set of 6 of those novels to read, with at least two by each author. How many different sets of 6 novels meet this criterion?

3. How many 5-card poker hands contain at least 3 face cards? [Remember, there are 12 face cards total: J, Q, K of each of the 4 suits.

4. Suppose you want to buy a dozen bagels and that the varieties you like are onion, parmesan, poppy seed, sundried tomato, and sunflower.

   (a) How many different "distributions" among these five varieties are possible? [For example, one distribution is 2 onion, 3 parmesan, 1 poppy seed, 4 tomato, and 2 sunflower; another is 3 onion, 4 parmesan, 0 poppy seed, 5 tomato, and 0 sunflower.]

   (b) How many different "distributions" are possible if you decide to have at least one of each of those varieties?

5. How many integers between 1 and 1000 inclusive are divisible by at least one of 3, 5, or 7? [Notational hint: let A be the set of numbers divisible by 3, B the set of numbers divisible by 5, and C the set of numbers divisible by 7.]

6. Suppose we have a set of 54 baseball players to be partitioned into 6 teams of 9 players each.

   (a) How many different ways are there to do this, if we don't care about the order of the teams, and if the players are not "specialized", so that *any* set of 9 players can comprise a team? Explain your counting method clearly.

   (b) Now suppose that the players are specialized to the following extent: there are 18 outfielders, 24 infielders, 6 pitchers and 6 catchers. Now how many ways are there to partition the players into teams, with each team consisting of 3 outfielders, 4 infielders, a pitcher and a catcher? Again, explain your counting method clearly.

7. Suppose that license plates consist of three letters (all upper-case) followed by three digits.

   (a) How many possible license plates are there?

   (b) How many license plates are there in which exactly one "A" appears?

   (c) How many license plates are there in which at least one zero appears?

   (d) How many license plates are there in which exactly one "A" *or* exactly one "8" (or both) appears?

(e) How many possible license plates would there be if one required that there be three digits and three uppercase letters, allowing repetitions, but allowed digits and letters to appear in any order (for example, A22Z3A would be an allowed plate).

8. There are 36 kids (18 boys and 18 girls) who want to play Little League.

   (a) How many ways are there to divide the Little Leaguers into four teams of nine players each? All that matters is the way the players are divided among the teams; the teams are not in any special order.

   (b) How many ways are there to select a "gender-balanced" team (i.e., 5 boys and 4 girls or 4 boys and 5 girls) from the pool of 36 players?

   (c) How many ways are there to field four gender-balanced teams for the Little League season? Hint: first explain why there must be exactly two 5 boy/4 girl and exactly two 4 boy/5 girl teams. Then solve the problem. As in the first part, there is no special order on the teams, but you need to remember that there are two different kinds of teams.

9. Suppose that there are 25 professors in the BSU math department, of whom 7 are statisticians and 12 are computer scientists.

   What is the largest number which could be the number of professors who are neither statisticians nor computer scientists? What is the smallest number?

Draw a Venn diagram illustrating each of the two extreme situations. Be sure to indicate in each of the two diagrams the number of statisticians, the number of computer scientists, the number of professors who fall in the intersection of the two categories, and the number who fall in neither category.

5. Suppose that thirty men and twenty women are on jury duty. If a twelve member jury is formed at random

   a. How many possible juries are there?

   b. How many possible juries are there with six men and six women?

10. Suppose that a fair six-sided die is rolled four times. How many different outcomes are possible? How many of the different outcomes involve rolling exactly four twos?

11. Suppose that all students at a certain technical college must take either biology, physics, or chemistry, or some combination of these courses. If there are 140 students in the class of 1992, and 80 took physics, of whom 40 took *only* physics, 60 took biology, of whom 20 took *only* biology, and 50 took chemistry, of whom 30 took *only* chemistry, and 10 students took both physics and chemistry, how many students took all three courses?

12. Of a group of students taking history, math, and chemistry, 9 got A's in history, of whom 4 had this as their only A, while 11 got A's in math, of whom 7 had this as their only A, and 12 got A's in chemistry, with 9 of these having this as their only A. 6 students didn't get any A's at all. 3 got A's in both history and math, 1 got A's in

math and chemistry, and 2 got A grades in both history and chemistry. How many students got an A in all three subjects? How many students were there in the whole group? Draw a Venn diagram...