Math 187 Assignment XI

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This problem set is due on Tuesday after spring break.

1. Is $\frac{1}{2560}$ expressible as a terminating decimal? You should be able to answer this question without consulting your calculator (indeed, your calculator may not be much help, depending on how many decimals it displays). If it is expressible as a terminating decimal, write down the exact decimal.

2. Determine the exact form of the fraction $\frac{1}{19}$ as a repeating decimal.

   Write a pair of fractions bounding $\frac{1}{19}$ above and below as closely as possible, both with denominator 100; then a pair of fractions bounding $\frac{1}{19}$ above and below as closely as possible, both with denominator 10000; then a pair of fractions bounding $\frac{1}{19}$ above and below as closely as possible, both with denominator 1000000.

3. Compute $\frac{2}{9}$ as an infinite repeating "decimal" in base 7. Don't forget that $9 = 12_7$.

   Using this infinite "decimal" in base 7, write pairs of fractional approximations bounding $\frac{2}{9}$ above and below more and more closely with denominators 7 in the first pair, 49 in the second, 343 in the third.

4. Write the number represented by $26.1235235235\ldots$ as a fraction (the block 235 repeats forever).

5. Write the number represented by $2.12121212\ldots$ in base 3 as a fraction. The block 12 repeats forever. Hint: the approach to be taken is exactly the same as in the previous problem, except for the difference of base.

6. Use the Euclidean algorithm to determine the "simplest form" of the fraction $\frac{5841}{4578}$.

7. Compute $\sqrt{10}$ to five digits using the algorithm given in class.

   Give pairs of fractions bounding $\sqrt{10}$ with denominators 100, 1000, 10000.

8. Verify that the definition of addition for our constructed rational numbers is independent of the choice of ordered pair (or "fraction") chosen to represent the rational number.

   The equivalence relation we use is defined thus: when a, c are integers and b, d are positive integers, $(a, b)\, R\, (c, d)$ is true if and only if $ad = bc$).

   We define addition thus: $[(a, b)] + [(c, d)] = [(ad + bc, bd)]$.

   (Remember that we abbreviate $[(a, b)]$ as $\frac{a}{b}$, and this looks much more familiar).

   Assume $(a, b)\, R\, (a', b')$ and $(c, d)\, R\, (c', d')$ (this is equivalent to assuming $[(a, b)] = [(a', b')]$ and $[(c, d)]\, R\, [(c', d')]$. Write out the condition in terms of the relation R needed for $[(a, b)] + [(c, d)] = [(a', b')] + [(c', d')]$ to hold, then verify that it follows from the assumptions. Your calculations should involve integers only.

9. If we want to check whether 10000001 is a prime by checking whether each prime $2, 3, 5, 7, 11 \ldots$ goes into it, we find quickly that $(11)(909091) = 10000001$. Now we want to continue the factorization by testing whether each prime $2, 3, 5, 7, 11, \ldots, p$ goes into 909091. State an accurate upper bound on the size of the largest prime p we might have to check before concluding that 909091 is prime (you don't have to find this prime exactly, just state a good approximation to where I should stop looking).

   A formal verification that 909091 is prime (it is prime!), including explicit checks whether 909091 is divisible by each prime that needs to be considered, is good for additional credit (you might want to use a computer or Maple to generate the check, but it is possible, though quite tedious to do it by hand with the help of a rather long list of primes).

10. Here's a useful factoring result generalizing theorem 8.6.1: if $a|bc$ and $\texttt{gcd}(a,b) = 1$, then $a|c$. Prove this. Hint: use the fact that a can be factored into primes and theorem 8.6.1.

11. Where a and b are integers, and $b|a$, we refer to the integer q such that $bq = a$ as $\frac{a}{b}$ as usual.

    Define $\texttt{lcm}(a,b)$ as $\frac{ab}{\texttt{gcd}(a,b)}$, for any positive integers a and b. This is the "least common multiple" of a and b.

    Show that $a|\texttt{lcm}(a,b)$ and $b|\texttt{lcm}(a,b)$. Hint: if you rewrite a and b in a suitable way, this becomes obvious.

    Show that if $a|m$ and $b|m$ (m is a multiple of both a and b) then $\texttt{lcm}(a,b)|m$. Explain why this result justifies the name "least common multiple".

    Hints: a can be written in the form $c\texttt{gcd}(a,b)$ for some c. b can be written in the form $d\texttt{gcd}(a,b)$ for some d. Why is $\texttt{gcd}(c,d) = 1$? You can use the result proved in the previous problem to solve this one (you are allowed to refer to it even if you have not succeeded in proving it). Also, don't be afraid to use fraction notation $\frac{a}{b}$ when you know that $b|a$: but don't write any "fractions" that you don't know to be integers.