Solutions

Math 187 Spring 2013, Test I

Dr. Holmes

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The exam begins at 9 am and ends at 10:15 am. You may use your writing instrument, your test paper and a non-graphing calculator. If you need scratch paper, I will have some.

in #3,
the original statement
is one of the four statements.

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1. Prove that $(A \to (B \to C))$ is logically equivalent to $(A \wedge B) \to C$ using the method of truth tables. Be sure to highlight relevant columns and say what fact about them verifies the logical equivalence.

A	B	C	A	→	(B	→	C)
T	T	T	T	T	T	T	T
T	T	F	T	F	T	F	F
T	F	T	T	T	F	T	T
T	F	F	T	T	F	T	F
F	T	T	F	T	T	T	T
F	T	F	F	T	T	F	F
F	F	T	F	T	F	T	T
F	F	F	F	T	F	T	F

(A	∧	B)	→	C
T	T	T	T	T
T	T	T	F	F
T	F	F	T	T
T	F	F	T	F
F	F	T	T	T
F	F	T	T	F
F	F	F	T	T
F	F	F	T	F

these are the same so the expressions are logically equivalent

2. Write an expression using only the "and", "or", and "not" operators (and the letters P, Q, R, of course) that has the following truth table. There is a purely mechanical way to do this from the table.

P	Q	R	???
T	T	T	F
T	T	F	F
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	T
F	F	F	F

$$(P \land \neg Q \land R) \lor \underbrace{(\neg P \land Q \land R) \lor (\neg P \land \neg Q \land R)}_{\neg P \land R}$$

equivalently

$$(\neg P \lor \neg Q) \land R$$

or

$$(\neg (P \land Q)) \land R$$

work

3. State and label the converse, inverse and contrapositive of the conditional statement "If it is snowing at 2000 feet, the temperature is below freezing at 10000 feet".

converse: If the temperature is below freezing at 10000 feet then it is snowing at 2000 feet.

inverse: If it is not snowing at 2000 feet, the temperature is not below freezing at 10000 ft

contrapositive: If it is not below freezing at 1000 ft, then it is not snowing at 2000 ft

Identify statement(s) on this list of four statements which are equivalent to the original statement and those which are not. Identify any other pairs of statements which are equivalent. You do not need to supply verification.

converse and inverse are equivalent to each other

the original statement is equivalent to the contrapositive

Identify which of these statements are true and which are false if it is not snowing at 2000 feet but it is freezing at 10000 feet.

The original and the contrapositive are true;

the converse and the inverse are false

4. Write the statement
"There is an integer x such that for every integer y, $x \leq y$." in quantifier notation (your notation should contain no English words at all).

~~$\forall x \in$~~ $(\exists x \in \mathbb{Z}. \; (\forall y \in \mathbb{Z}. \; x \leq y))$

Write the negation of this statement in a form in which no negation symbol appears (use de Morgan laws and rules for negation of quantifiers as far as possible, then change the order relation appropriately).

$(\forall x \in \mathbb{Z}. (\exists y \in \mathbb{Z}. \; x > y))$

Translate the sentence from part b into an English sentence in which no mathematical notation appears (there is a natural way to say it without even using letters).

For each integer, there is a smaller integer.

5. Counting problems

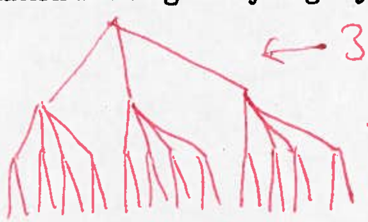
(a) Factor 600 into primes. Determine the total number of positive factors of 600 (both prime and composite; there are quite a few). Show a calculation on your paper which makes it clear that you know the trick for solving this problem.

$600 = 2 \cdot 3 \cdot 2^2 \cdot 5^2 = 2^3 \cdot 3 \cdot 5^2$

has $(3+1)(1+1)(2+1) = 4 \cdot 2 \cdot 3 = 24$ factors

(b) At an ice cream bar you may take one scoop of ice cream, either chocolate, vanilla, or strawberry. There are three kinds of sauce, hot fudge, caramel, and butterscotch, and you have the option of putting nuts on your sundae. You are allowed to leave out the sauce and/or the nuts. How many different kinds of sundaes can you make?

Draw a tree diagram (or enough of it to give the idea, its pretty big) and a calculation showing how you got your answer.



← 3 ice cream choices

← 4 topping choices including none

← 2 nuts choices (Y/N)

$3 \cdot 4 \cdot 2 = 24$ possible sundaes

(c) A state issues license plates with three letters followed by four digits. How many plates are possible if no letter may appear more than once on the plate and no digit may be immediately followed by the same digit? Show a calculation and the final answer.

$26 \cdot 25 \cdot 24 \cdot 10 \cdot 9 \cdot 9 \cdot 9$

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6. Prove that the sum of two odd integers is even. First restate it in a form which makes its logical structure as an implication clear, then prove it. You should know our official definitions of "odd" and "even" by now. Remember that there are places where you need to say or confirm that certain numbers are integers.

~~Let x~~

~~Assume~~ p

If x and y are odd, then $x+y$ is even.

Assume that ① x is odd ② y is odd

Goal: Show that $x+y$ is even.

by ① there is an integer m s.t. $x = 2m+1$

by ② there is an integer n s.t. $y = 2n+1$

Rewrite Goal: Find an integer p s.t. $x+y = 2p$

$x+y = (2m+1)+(2n+1) = 2m+2n+2 = 2(m+n+1)$

Let $p := m+n+1$ ~~this~~ p is an integer by closure

$x+y = 2p$ and we are done.

7. **Formal proof.** Using the rules in the style manual, prove

$$((P \to Q) \wedge (Q \to R)) \to (P \to R).$$

Assume ① $(P \to Q) \wedge (Q \to R)$

Goal: $P \to R$

Assume ② P

Goal: R

③ $P \to Q$ simp, 1

④ $Q \to R$ simp, 1

⑤ Q m.p. 2,3

6 R m.p 4,5

which is what we needed.

⑦ $P \to R$ 2-6 deduction

optional ⑧ $((P \to Q) \wedge (Q \to R)) \to (P \to R)$ 1-7 deduction

1 Brief logical style manual

I'm providing the rules for proving and using conjunctions and the rules for proving and using implications, all for direct proofs (no mention of negation).

Proving a conjunction: To prove a conjunction $A \wedge B$, first prove A, then prove B.

Using a conjunction (simplification): If you are entitled to assume $A \wedge B$, you are also entitled to assume A. (simplification).

If you are entitled to assume $A \wedge B$, you are also entitled to assume B (simplification).

Proving an implication: To prove $P \to Q$:

Assume: P

Goal: Q

...proof steps...

last line... Q

Using an implication: If you are entitled to assume A, and you are entitled to assume $A \to B$, then you are further entitled to assume B. This is the rule of modus ponens.