Math 187 Test II, Spring 2012

Dr. Holmes

March 9, 2012

The exam begins at 1:40 pm and ends at 2:35. You are allowed the use of your test paper, your writing instrument and your non-graphing calculator.
1. Draw a Venn diagram illustration of the identity

\[ A - (B \cap C) = (A - B) \cup (A - C). \]

Your two diagrams (one for each side of the equation) should show appropriate shadings so I can understand how you obtained your sets), and the result set should be outlined on each diagram (so I can see that the two result sets are the same).
2. Draw Venn diagrams illustrating the fact that

\[ A \cup (B \cap C) = (A \cup B) \cap C \]

is not a theorem, then exhibit actual sets in list notation for which this equation is not true.

The same instructions about shading and outlining result sets apply to the diagrams.
3. Some counting. Show the setup of your calculation using factorials, falling factorials and/or binomial coefficients then compute the answer. Do two of the three parts; if you do all three your best work will count.

(a) In how many ways can the letters of REARRANGED be rearranged? Show the setup of your calculation using factorials then compute the answer.

(b) In how many ways can 12 children be divided into four teams of three, called the Angels, the Bears, the Cats and the Dogs? In how many ways can 12 children be divided into four unnamed teams?

(c) How many five digit numbers can be formed which contain no zeros (so we don’t have to worry about not starting with 0) and exactly three 1’s? Hint: first place the 1’s, then choose the other numbers.
4. More counting. Show the setup of your calculation using factorials, falling factorials and/or binomial coefficients then compute the answer. Multiset counting symbols must be converted to binomial coefficients then evaluated.

(a) In this lettered part, please show how to compute any binomial coefficients you use using multiplication and division.
A committee with 15 members is to choose a subcommittee with 4 members. How many ways are there to do this?

A committee with 15 members is to choose a subcommittee with 4 members, with a member designated as chair and a member designed as secretary. How many ways are there to do this?

(b) From a bucket of Scrabble tiles in which there are at least 7 of each letter, how many ways are there to draw 7 letters and arrange them on your letter rack in order?

How many ways are there to draw a handful of 7 tiles from the bucket (hint: this is a multiset problem)?
5. In a group of 24 students, 8 take Math, 11 take French and 14 take English. 3 take Math and English. 4 take English and French. 4 take Math and French. Every student is taking at least one of these courses. How many are taking all three courses? Your work should show that you know how to use the Inclusion-Exclusion Principle.

Write out the inclusion exclusion formula for $|A \cup B \cup C \cup D|$ in terms of the sizes of $A$, $B$, $C$, $D$, and all intersections of two, three or four of these sets.

Suppose that sets $A, B, C, D$ each have 50 members. Each intersection of two of the sets has 10 members. Each intersection of three of the sets has 3 members. There is 1 member of all four sets. How many elements does $A \cup B \cup C \cup D$ have?
6. Properties of relations. Do one of the two parts. If you do both, your best work will count.

(a) Another person who shares both mother and father with you is your full sibling. Is the relation \( x S y \) meaning \( x \) is a full sibling of \( y \) reflexive? Is it symmetric? Is it transitive? What you write in reply to this should make it clear that you know what “reflexive”, “symmetric”, and “transitive” mean.
(b) Make a Hasse diagram of the partial order $m|n$ ($m$ goes into $n$) on the natural numbers $n$ such that $1 \leq n \leq 12$. Identify the minimum element if any, minimal elements, the maximum element if any, and maximal elements of this partial order.

Recall that in a Hasse diagram you do not have to draw all arrows, because you assume that the relation is reflexive, antisymmetric and transitive, and arrows whose presence can be deduced from these properties and the arrows shown do not need to be drawn.
7. Do one of the two induction proofs. If you do both, your best work will count. Your proof needs to be properly structured, with a clearly identified basis step, induction hypothesis and induction goal. Be sure to show me where in your proof of the induction goal you use the induction hypothesis.

(a) Prove by mathematical induction that the sum of the first \( n \) odd numbers is \( n^2 \):

\[
\sum_{i=1}^{n} (2i - 1) = 1 + 3 + 5 + \ldots + (2n - 1) = n^2
\]
(b) Prove by mathematical induction that $n^3 + 5n$ is divisible by three for each natural number $n$. 