Math 187 Test Two, Spring 2010

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This test will begin at 2:40 pm and end at 3:30 pm. You may use a standard scientific calculator but not a graphing calculator. Cell phones must be turned off and out of sight. The question you do worst on will not count; if you do all questions your best work will count.
1. Write an illustration using Venn diagrams of the identity

\[ A \cup (B \cap C) = (A \cup B) \cap (A \cup C). \]

Be sure to use shadings appropriately in each diagram, with a key, and be sure to highlight the set which is the final result on each side of the diagram.
2. The equation

\[ A \cup (B - C) = (A \cup B) - (A \cup C) \]

is not correct in general. Give a Venn diagram illustration of the fact that this is not correct, and use your diagram to construct actual sets \( A, B \) and \( C \) for which this equation is false. Write your counterexample sets out in list notation.
3. At a small school there are 23 freshmen. Every one of them must take at least one course from Math, English, French. 14 of them are taking English. 15 of them are taking Math. 11 of them are taking French. 5 of them are taking both English and French, 5 of them are taking both Math and French. 11 of them are taking both English and Math. How many of the students are taking all three subjects?

I remind you of the useful equation

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|.$$
4. Give sets $A, B, C$ in list notation for which $A \cap B \cap C = \emptyset$ and $|A| + |B| + |C| \neq |A \cup B \cup C|$. Compute the two quantities for your given sets and show that they are different.
5. Write out the following sets in list notation.

(a) \[ \{x \in \mathbb{N} : 3| x \land x|24\} \]

(b) \[ \{A : A \subseteq \{1, 2, 3\} \land |A| = 2\} \]

(c) \[ \{1, 2, 3\} \times \{2, 5\} \]
6. Relations and relation properties – basics

(a) Write the relation \( x|y \) (\( x \) goes evenly into \( y \), \( y \) is divisible by \( x \)), restricted to the set \( \{1, 2, 3, 4, 5, 6\} \), as a set of ordered pairs.
Draw a picture of this relation using arrows. Show all arrows, including loops.

(b) Present a relation (you get to pick one) on the set \( \{1, 2, 3, 4\} \) which is not transitive. You may present your relation using an arrow diagram or using list notation for a set of ordered pairs. Explain why it is not transitive. Your explanation should reveal an understanding of the definition of “transitive” and it should mention specific numbers.
7. (a) You are going to start explaining why the relation on human beings \( x B y \) defined by \( x \) has the same birthday as \( y \) is an equivalence relation. Write out the three things you need to verify. Your statements should not include the words “reflexive”, “symmetric”, or “transitive”, or the symbol \( B \) for the relation: expand out all definitions. Your statements can use letters \( x, y, z \) to represent people. You do not need to prove these statements: in fact each of them is obviously true.

\( B \) is reflexive:

\( B \) is symmetric:

\( B \) is transitive:
(b) The relation \( x T y \) defined by “\( x - y \) is divisible by three”, restricted to the set \( \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\} \), is an equivalence relation.

Write the equivalence class [8].

List all the equivalence classes.
8. In each part you need to show a calculation using basic arithmetic operations (addition, subtraction, multiplication, division: if you have permutations and combinations on your calculator, your result won’t count unless you know how to compute it using basic operations) as well as a final answer. You can write factorials and powers: I assume you know how to get these using multiplication.

(a) In how many ways can we rearrange the letters of the word NEEDLE?

(b) In how many ways can we rearrange the letters of the word PEBBLE?

(c) A subcommittee with three members is to be chosen from a committee with seven members. In how many ways can this be done?