Math 187 Test II

Dr. Holmes

This test begins at 8:40 am and ends at 9:35. You are allowed to use a plain scientific calculator with no graphing or symbolic computation capability. Cell phones must be turned off and out of sight.

1. Give a demonstration of the theorem

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

   using Venn diagrams. Provide keys to your diagrams and clearly indicate the sets which are the final answers.

2. Draw the Hasse diagram for the relation of divisibility $(x|y)$ restricted to the set of all positive integers less than or equal to 12. Draw no more arrows than are needed.

   Identify a minimum element for this partial order or say that it doesn't have one. Identify all minimal elements for this partial order or say that it doesn't have any. Identify a maximum element for this partial order or say that it doesn't have one. Identify all maximal elements for this partial order.

3. Write down the equivalence classes for the equivalence relation $x\,R\,y$ defined as "$x - y$ is divisible by 4" on the set of positive integers less than or equal to 20 (these are sets).

4. In how many different ways can the letters in REARRANGED be rearranged? Set up your answer in terms of factorials then compute an actual number.

5. You have a bin containing 12 beads of each of 4 different colors. You are going to make an open chain of just 12 beads (not 48). In each part set up your answer in terms of powers, factorials or binomial coefficients (as appropriate) then compute an actual number.

   (a) How many different chains of 12 beads with 3 of each color can you make?

   (b) How many handfuls of 12 beads of assorted colors are possible (here all that matters is how many beads you have of each color)?

   (c) How many chains of 12 beads of any mixture of colors are possible?

6. A committee of 10 people is appointing a subcommittee of 5 people. In each part, set up your calculation using binomial coefficients then compute an actual number. The last part is of course unrelated, but you still need to show your answer first using binomial coefficients then using actual numbers.

   (a) How many subcommittees of 5 are possible?

   (b) Suppose we also need to specify a chair and a secretary from among the members of the subcommittee. How many possible ways are there to choose a 5 member subcommittee with chair and secretary?

   (c) Compute the first four terms of the expansion of $(x + y)^{15}$.

7. Of a group of 26 students, all must take English, Math, or French. 12 take English, 14 take Math, 14 take French. 7 take English and French, 3 take English and Math, 6 take French and Math. How many are taking all three subjects? You must set up and solve the problem in a way which demonstrates that you understand the Inclusion-Exclusion Principle.

8. Proofs. Do one of the following: if you do both, you will be graded on your best work. If you do very well on both you might get some extra credit.

   (a) Prove that the relation $x\,R\,y$ on integers defined by "$x - y$ is dvisible by 4" is an equivalence relation. There are three things to prove: set them up and identify each with an appropriate name, then prove them.

(b) Prove using the appropriate proof templates from the book that for any sets A, B, C, if $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$. Of course this is obvious, and you do not get noticeable credit for an informal explanation of the obvious; what is wanted is a formal proof in the style of the book or my board examples.