Math 187 Test I

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This exam begins at 8:40 am and ends at 9:35 pm. You may use a calculator with no graphing or symbolic computation capabilities (which may not be a cell phone or PDA). Cell phones must be turned off and out of sight; no exceptions.

You may omit one numbered question. If you do all questions your best work will be graded.

1. Show that a conditional $A \to B$ is logically equivalent to its contrapositive $\neg B \to \neg A$, by setting up an appropriate truth table and highlighting and commenting on appropriate columns of the table.

   Show that a conditional $A \to B$ is *not* logically equivalent to its inverse $\neg A \to \neg B$. Discuss a specific assignment of truth values to A and B which demonstrates this.

2. Write down an expression using only letters, $\wedge$, $\vee$, and $\neg$ which has the truth table shown. If you do any complicated calculations, you are on the wrong track: this is just to be read off the table.

P	Q	R	???
T	T	T	F
T	T	F	T
T	F	T	F
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	T
F	F	F	F

3. License plates for a small state consist of three letters $(A - Z)$ followed by three digits $(0 - 9)$. Show the setup for your calculations and give the final answer if you have a calculator.

   (a) How many license plates are possible if no other conditions are imposed?

   (b) How many license plates are possible if no letter can appear more than once on the plate and no digit can be immediately followed by the same digit?

   (c) Allowing unlimited repetitions, how many license plates contain at least one letter A?

4. Demonstrate that the statement "For any integers x and y, if $6|xy$ then either $6|x$ or $6|y$" is false by giving a counterexample and a full explanation of why it is a counterexample.

5. Negations

(a) Rewrite $\neg(A \vee (B \wedge C))$ in a form in which negations are applied only to single letters, using de Morgan's laws.

(b) Rewrite the symbolic expression "$\forall x \in \mathcal{Z}, \exists y \in \mathcal{Z} x + y = 0$" in mathematical English (English with variables).

Rewrite the negation "$\neg(\forall x \in \mathcal{Z}, \exists y \in \mathcal{Z}, x + y = 0)$" of this symbolic expression with the negation moved to the right of the quantifiers.

Express the negated sentence in natural mathematical English.

6. Set theory notation: in each of the following sentences, write $\in$ (is a member of) or $\subseteq$ (is a subset of) as appropriate.

   (a) 2 ___ $\{1, 2, 3\}$
   (b) $\emptyset$ ___ $\{1, 2, 3, 4\}$
   (c) $\{1, 2\}$ ___ $\{1, 2\}$
   (d) $\{1\}$ ___ $\{\{1\}\}$

7. Show that the proposition $P \to (Q \to R)$ is logically equivalent to the proposition $(P \wedge Q) \to R$ by setting up a truth table and highlighting and commenting on appropriate columns of the table.

8. Prove that the sum of two odd integers is even, in the style we have studied in class, using the standard definitions we have worked with in class. Start by reexpressing the theorem as a conditional, introducing variables as needed.

9. Prove that for any integers a, b, and c, if $a|b$ and $a|c$ then $a|(b+c)$, in the style we have studied in class, using the standard definitions we have worked with in class.