

# Math 187 Final Exam

Dr. Holmes

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This exam starts at 8 am and officially ends at 10 am. If no one objects, the exam will continue until 10:15 am. You are allowed to use your book and one standard sized sheet of notebook paper with whatever you like written on it. You are allowed to use a standard scientific calculator without graphing or symbolic computation capabilities.

1. Show that  $P \rightarrow (Q \rightarrow R)$  is logically equivalent to  $(P \wedge Q) \rightarrow R$  by the method of truth tables. Be sure to highlight significant rows or columns of your tables and state any important relationships between them.

2. Give a formal proof that the product of two odd integers is odd. Your proof should use nothing but the definition of “odd” (which you should state) and basic algebra. It should be given in the formal style with assumptions and goals which was taught in class.

3. A state has license plates consisting of four letters followed by three digits. Set up your calculations on your paper then compute the numerical answer.
- (a) How many plates are possible?
  
  
  
  
  
  
  
  
  
  
  - (b) How many plates are possible if no letter can appear more than once on the plate and no digit can be immediately followed by the same digit?
  
  
  
  
  
  
  
  
  
  
  - (c) How many plates contain no digit 8?
  
  
  
  
  
  
  
  
  
  
  - (d) How many plates have no repeated letters or digits, all letters in alphabetical order and all digits in numerical order? (It is still the case that the letters all appear before the digits.)

4. In a group of 32 freshmen, all of whom must take at least one of English, Math, and French, 16 students take English, 19 take Math, 18 take French, while 9 take English and French, 10 take Math and French, and 10 take English and Math. How many take all three courses?

You need to set up and carry out your work using the Inclusion Exclusion Principle; I should be able to tell from your paper that you know what this principle is, at least in the case of three sets.

5. Prove by mathematical induction that for any nonnegative integer  $n$ , the sum  $\sum_{i=0}^n 3^i = 1 + 3 + 9 + \dots + 3^{i-1} + 3^i = \frac{3^{n+1}-1}{2}$ .

6. Compute the gcd of 30 and 74 using the Euclidean algorithm (and clearly indicate that you know what the answer is:  $\gcd(30, 74) = \dots$ ).  
Find integers  $x$  and  $y$  such that  $30x + 74y = \gcd(30, 74)$ .

7. Chinese Remainder Theorem.

Solve the system of equations

$$x \bmod 23 = 11$$

$$x \bmod 34 = 2$$

Give the smallest non-negative solution.

8. Degree sequences: for each of the following degree sequences either present a graph with that degree sequence or give a reason why no such graph is possible. Also, follow any additional instructions in each part.

(a) 1,2,2,3,3,3

(b) 1,2,3,4,4,5

(c) 2,2,2,2,2,2 (there are at least two fundamentally different graphs with this sequence: draw two).

9. A graph has 8 vertices, each of degree 5. I drew one: this is possible. You should not have to draw one to answer the questions, however. How many edges are there in this graph? Explain.

Show that the graph is not planar using an appropriate formula.

10. A graph is pictured.

Identify all the cut edges in this graph (put an x in the middle of each cut edge on the given graph picture).

Draw a spanning tree for this graph (a spanning subgraph which is a tree: draw a separate picture of a subgraph of the given graph with the same vertices which is a tree).