

Math 187 Test II with some solutions

Dr. Holmes

This exam will begin at 1:40 am and end at 2:35 am. You may use a standard scientific calculator with no graphing or symbolic computation capabilities. Please note that setup of computations of binomial coefficients or falling factorials will be requested, so don't rely on any built in functions your calculator may have (using them to check is fine). Books, notes and neighbors should remain firmly closed.

1. No solutions for this problem, it was on Test I

License plates in a certain state consist of three letters followed by four digits.

- (a) How many license plates are possible?

- (b) How many licence plates are possible if no letter or digit may be repeated on the plate?

- (c) How many license plates are possible if no letter or digit may be immediately followed by the same letter or digit?

- (d) How many license plates contain no A 's and no 8 's?

- (e) How many license plates contain at least one A or at least one 8 ?

2. (a) Give a demonstration of the identity $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ using Venn diagrams. Be sure to use shadings appropriately in each diagram, with a key, and be sure to highlight the set which is the final result on each side of the diagram.
- Can't do pictures easily online.

- (b) The equation $A - (B - C) = (A - B) - C$ is in general false. Give a pair of Venn diagram illustrating the fact that it is false and write explicit finite sets (listing their elements) which are counterexamples. You can use your Venn diagram to help develop the counterexample.

I cant do Venn diagrams here, but I can produce a counterexample.

$$\{1, 2\} - (\{2, 3\} - \{2, 4\}) = \{1, 2\} - \{3\} - \{1, 2\}$$

$$(\{1, 2\} - \{2, 3\}) - \{2, 4\} = \{1\} - \{2, 4\} = \{1\}$$

3. There are 24 freshmen at a very small private high school. Every one of them takes at least one of Math, English, and French. 14 of them take English. 16 of them take French. 6 of them take both English and Math. 7 of them take both Math and French. 7 of them take both English and French. 4 of them take all three subjects.

How many of them take Math?

$$|M \cup E \cup F| = |M| + |E| + |F| - |M \cap E| - |M \cap F| - |E \cap F| + |M \cap E \cap F|$$

that is

$$24 = x + 14 + 16 - 6 - 7 - 7 + 4$$

from which algebra gives $x = 10$.

You can also solve this pretty easily by filling in spaces in a Venn diagram with numbers. I did this in class.

Solve this problem in a way that makes it clear that you understand the application of the Inclusion-Exclusion principle to three sets.

4. How many ways are there to rearrange the letters in the word CHANGE? How many ways are there to rearrange the letters in the word REARRANGE? Be sure to show your setup on your paper, not just show a final answer.

In CHANGE, $6! = 720$

In REARRANGE

$$\frac{9!}{(3!)(2!)(2!)} = 15120$$

5. Binomial coefficients

- (a) In how many ways can a committee with 20 members choose a subcommittee with four members? Show the complete details of the computation of this number from basic operations.

There are $(20)(19)(18)(17)$ ways to list 4 of 20 members in order. There are $4!$ ways to list the members of a committee with 4 members, so there are

$$\frac{(20)(19)(18)(17)}{4!} = 4845$$

possible committees.

- (b) In how many ways can nine children be divided into three teams named the Lions, the Tigers, and the Bears?

$$\frac{9!}{(3!)(3!)(3!)} = 1680$$

– its the same as the number of ways to rearrange the letters in the word LLLTTTBBB. We did this in class.

In how many ways can three children be divided into three teams with three members each (where we have no interest in which team is which)? Show complete setup for these calculations on your paper.

The number in the previous problem divided by three: think about renaming the teams: if you changed the Lions into Tigers and the Tigers into Lions you would still have the same three teams. There are six ways to assign the three names to the three teams, so there are one-sixth as many divisions into three teams as there are assignments of the students to named teams.

- (c) Write out the expansion for $(x + y)^5$ using explicit binomial coefficients, then write it with the coefficients evaluated.

Not on this test.

6. Relations

- (a) Write the relation $x \leq y$ on $\{1, 2, 3\}$ as a set of ordered pairs. Make sure you use correct notation.

$$\{(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)\}$$

- (b) Consider the relation “ A is a sibling of B ” on human beings (a sibling is a full brother or sister).

Is this relation reflexive? Explain. An important part of the explanation is to make it clear that you know what “reflexive” means.

No, for no x is it true that x is the brother or sister of x .

Is this relation symmetric? Explain. An important part of the explanation is to make it clear that you know what “symmetric” means.

Yes, if x is the brother or sister of y then y is the brother or sister of x .

Is this relation transitive? Explain. An important part of the explanation is to make it clear that you know what “transitive” means.

7. Write the following sets by listing all their elements between braces.

(a) $\{x \in \mathbb{N} \mid 2|x \wedge x|12\}$

$$\{2, 4, 6, 12\}$$

(the set of all x which are even and also factors of 12).

(b) $A \times B$, where $A = \{1, 2\}$ and $B = \{3, 4, 5\}$.

$$\{(1, 3), (2, 3), (1, 4), (2, 4), (1, 5), (2, 5)\}$$

(c) The power set of $\{1, 2\}$ (i.e., the collection of all subsets of $\{1, 2\}$)

$$\{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$$

(d) $\{A \mid A \subseteq \{1, 2, 3, 4\} \wedge |A| = 2\}$

The set of subsets of $\{1, 2, 3, 4\}$ with two elements:

$$\{\{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}\}$$

8. Equivalence relations

Prove that the relation $x R y$ is an equivalence relation, where $x R y$ is defined as “ $x - y$ is even”. There are three things to prove: state them, clearly identifying each of them, and prove them.

Reflexive: You need to prove that $x R x$: this means $x - x$ is even, which is true because 0 is even.

Symmetric: You need to prove that if $x R y$ then $y R x$, that is if $x - y$ is even then $y - x$ is even. If $x - y = 2k$ then $y - x = 2(-k)$, so if the first number is even so is the second.

Transitive: You need to prove that if $x R y$ and $y R z$ are even then $x R z$ is even: if $x - y = 2m$ and $y - z = 2n$ then $x - z = (x - y) + (y - z) = 2m + 2n = 2(m + n)$, so if the first two numbers are even so is the third.

List the equivalence classes under this relation if it is restricted to the set $\{1, 2, 3, 4, 5, 6, 7, 8\}$.

$$\{1, 3, 5, 7\}, \{2, 4, 6, 8\}$$

Draw pictures of all the possible equivalence relations on the three-element set $\{a, b, c\}$. You may illustrate them using arrow diagrams or partitions.

Im using parentheses instead of circles to picture partitions since I cant draw

(abc) $(ab)(c)$ $(ac)(b)$ $(bc)(a)$ $(a)(b)(c)$