Math 187 Final Exam (Fall 2008)

Dr. Holmes

This exam begins at 1 pm and ends officially at 3 pm. If everyone agrees, the exam can continue until 3:15 pm. You are allowed to use your book, a standard scientific calculator without graphing or symbolic computation capability, and loose sheets of paper (notes and paper on which to write your answers). You are not allowed bound notebooks or photocopied handouts other than the exam paper. Please write your answers on your own paper. On each page, clearly indicate who you are and what problem you are working on. Please work different problems on different pages. When you return your paper, be aware that you need to return this exam paper and the pages on which you have recorded your answers, and please arrange the pages in order of problem number when you hand them back.

Good luck!

1. Show that \((P \land Q) \to R\) is logically equivalent to \((P \to (Q \to R))\) using the method of truth tables. Be sure to highlight relevant rows and/or columns and say in English what is relevant about them.

2. The sets of ordered pairs \(f = \{(1, 3), (2, 4), (3, 1), (4, 5), (5, 2)\}\) and \(g = \{(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)\}\) represent permutations of the set \(\{1, 2, 3, 4, 5\}\). Write each of the permutations in table notation and cycle notation, then write the composition \(f \circ g\) in table and cycle notation.
3. Counting problems. This problem is a mandatory question on this exam and also a makeup for the similar problem on an earlier exam. For each part of this question, indicate whether the counting technique used allows for repetition of items and whether order of items matters. Also, answer the question and compute the answer (don’t just set up a calculation: you must both set it up and carry it out).

(a) In how many ways can I make a chain of red, green, and blue beads if the chain is to have five beads and I have an unlimited supply of beads of each color?

(b) A committee with ten members must choose a subcommittee with three members. In how many ways can this be done?

(c) You want to order ten bagels. The flavors available are plain, garlic, onion, poppy seed, and sesame. How many bagel orders are possible?

(d) I have one Scrabble tile of each of the vowels AEIOUY. In how many different ways can I form a “word” of three vowels on my Scrabble tray?

4. Give a formal proof that the product of two odd numbers is odd. You may use our official definitions of “odd” and “even” and basic arithmetic and algebra. You may NOT use facts about divisibility as that is the sort of thing you are proving here! Set up your proof in the format taught in class.

Definitions provided: an integer $a$ is even provided that there is an integer $x$ such that $2x = a$; an integer $b$ is odd provided that there is an integer $y$ such that $b = 2y + 1$.

5. Of 29 housewives at the registers at Albertson’s one morning, 14 bought bread, 19 bought milk, and 16 bought cheese. 6 bought both bread and milk; 11 bought both milk and cheese; 7 bought bread and cheese. Determine how many bought all three. Your work should show that you understand the Inclusion/Exclusion principle for three sets.
6. Prove by mathematical induction that
\[ \sum_{i=1}^{n}(2i - 1) = n^2 \]
that is,
\[ 1 + 3 + 5 + \ldots + (2n - 3) + (2n - 1) = n^2 \]
for each natural number \( n \), or equivalently the sum of the first \( n \) odd numbers is \( n^2 \).
In your proof you should clearly label the basis step, the induction hypothesis, the goal of the induction step, and the place or places where the induction hypothesis is used in your proof.
There are proofs or sketches of proofs of this statement in the book, but not a proper mathematical induction proof (copying them will do you no good). Correct format is a lot of what I'm interested in here.

7. Compute the gcd of 27 and 64 using the Euclidean algorithm. Determine integers \( x \) and \( y \) such that \( 64x + 27y = \gcd(64, 27) \).
Fill in the blanks in this form:
\[ \___ = \gcd(36, 25) = 36*\___ + 25*\___ \]
Then solve the equation
\[ 27x \equiv 11 \mod 64 \]

8. Solve the system of equations. Give the smallest non-negative solution.
\[ x \equiv 10 \mod 31 \]
\[ x \equiv 21 \mod 37 \]
9. Give the group operation table for \((\mathbb{Z}_7^\ast, \odot)\), that is, multiplication in mod 7 arithmetic restricted to elements relatively prime to 7.

State what the identity of this group is.

State what the inverse is of each element of the group.

State the order of each element of the group and identify a generator of the group.

Give an isomorphism between this group and the addition group of mod 6 arithmetic.

10. Two graphs are pictured. One of them admits an Eulerian walk and one does not. Explain why the one that does not, does not, and give an Eulerian walk in the other as a list of vertices visited in order (make sure you cover all edges in the graph!)