

Math 187 Test III

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This test will begin at 1:40 pm and end at 2:35 pm. You may use (and will need) a plain scientific calculator without graphing or symbolic computation capabilities. You may omit one numbered question: if you do all questions, your best work will count. Problem 8 is extra credit, over and above that (do not attempt it unless you've done everything you can on the rest of the exam).

1. The sequence G_n is defined by the recurrence relation

$$G_0 = 2; G_1 = 3; G_n = 2G_{n-1} + 8G_{n-2} \text{ for } n > 1$$

Compute the next four terms of the sequence (up to G_5) using the recurrence relation.

Find a closed form expression for G_n (reminder: you will use a quadratic). Show all work. Verify that you get the correct value for G_5 .

2. How many functions are there from $A = \{1, 2, 3\}$ to $B = \{1, 2, 3, 4, 5\}$?

How many of them (if any) are one-to-one? How many of them (if any) are onto?

Give one of these functions (in the form of a list of ordered pairs) which is neither one-to-one nor onto B . Explain in detail why it is not one-to-one (mentioning specific elements of the set A) and why it is not onto B (mentioning a specific element of the set B).

How many functions are there from $C = \{1, 2, 3, 4\}$ to $D = \{1, 2\}$?

How many of them (if any) are one-to-one? How many of them (if any) are onto?

Give one of these functions (in the form of a list of ordered pairs) which is neither one-to-one nor onto D . Explain in detail why it is not one-to-one (mentioning specific elements of the set C) and why it is not onto D (mentioning a specific element of the set D).

3. Fancy counting concepts

- (a) A square with side length 4 is divided into 16 smaller squares of side length 1 in the obvious way. 19 points are chosen in the interior of the large square. Explain why two of them must be have distance $\leq \sqrt{2}$ from each other.

Your explanation will use the Pigeonhole Principle. Make sure that the reader can tell that you understand what the Pigeonhole Principle is and why it applies here. Make sure that you explicitly mention any geometric facts you need.

- (b) Describe a bijection from the natural numbers to the integers. An informal arrow diagram that makes the pattern clear is sufficient.

4. Functions f and g are given as lists of ordered pairs.

$$f = \{(1, 2), (2, 2), (3, 4), (4, 5)\}$$

$$g = \{(2, 2), (3, 5), (4, 3), (5, 4)\}$$

State the domain and image of f as sets.

State the domain and image of g as sets.

For each of f^{-1} and g^{-1} , state whether it is a function or not: if it is not a function, explain (mentioning specific domain or image elements in your explanation) and if it is a function, present it as a list of ordered pairs.

Determine $g \circ f$ as a set of ordered pairs or explain why it does not exist. An arrow diagram might help.

Determine $f \circ g$ as a set of ordered pairs or explain why it does not exist. An arrow diagram might help.

5. Two permutations in S_6 are presented, one in array notation and one in cycle notation.

$$\sigma = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 1 & 3 & 6 & 5 \end{bmatrix}$$

$$\tau = (145)(236)$$

Express σ in cycle notation and τ in array notation.

Express each of τ^{-1} and $\sigma \circ \tau$ in whichever notation you prefer.

Write down a permutation ρ in S_{10} (cycle notation is probably best for this) with the property that $\rho^{21} = \iota$. (ι is the identity permutation which sends each number to itself). It is no fair using the identity permutation as ρ . Hint: choose the lengths of cycles in your permutation carefully.

6. Compute the greatest common divisor of 2701 and 629. Show all steps. Clearly identify the number which you claim is the gcd! Use your gcd calculation to find integers x and y such that $2701x + 629y = \gcd(2701,629)$ (and include a check that they work).

7. Write down the multiplication table for arithmetic mod 7.

Determine the multiplicative inverse of 21 in mod 37 arithmetic using a gcd calculation.

8. **Extra Credit** (do not attempt unless you have done everything you can on the rest of the exam): Prove that if $f : A \rightarrow B$ and $g : B \rightarrow C$ are one-to-one functions, then the composition $g \circ f$ is a one-to-one function.