Math 187 Test II

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You have from 1:40 pm to 2:35 pm to work on this exam. You may use a plain scientific calculator with no graphing or symbolic computation capability. When computing falling factorials or binomial coefficients, you must indicate how to compute these using multiplication and division, even if your calculator has built-in permutations and combinations (some do).

1. (a) How many anagrams can be made from the word ABRACADABRA? Set this up in terms of factorials then compute it.

   (b) A chain of beads is to be made with 12 beads, 4 beads of each of 3 different colors. How many different ways are there to do this? Set this up and compute it.

2. (a) A committee with 20 members is to choose a subcommittee with 5 members. How many ways are there to do this? (Set up and compute actual number).

(b) A committee with 20 members is to choose a committee with 5 members which will have a chair and a secretary. How many ways are there to do this? (Set up and compute actual numbers).

(c) A committee of 12 members is to divide itself into 3 working groups each with 4 members. How many ways are there to do this if the groups are labelled Rules, Membership, Finance? How many ways are there to do this if the groups have no special identifiers? Set up and compute actual numbers.

3. Write the first four terms of the expansion of $(x+y)^{20}$. Set this up using binomial coefficients, then evaluate the coefficients as actual numbers.

4. As is so often the case in my tests and examples, members of a group of 21 students are all enrolled in at least one of Math, French and English. 10 students take Math. 9 students take French. 13 students take English. 3 students take Math and French. 6 students take English and Math. 4 students take English and French.

   Either compute the number of students taking all three courses or prove that I am lying. Your solution must use the inclusion-exclusion principle: set up your work in such a way that I can clearly see that you used it.

   Write the inclusion-exclusion formula for $|A \cup B \cup C \cup D|$, where A, B, C, D are any four sets.

5. How many ways are there to order four scoops of ice cream using Baskin-Robbins's famed 31 flavors? The order of the scoops does not matter (the answer is not 31^4) and you are permitted to order more than one scoop of the same flavor.

6. Consider the relation $x\,F\,y$ on integers defined as $5|(x-y)$.

   Write out the three things you need to prove to show that this is an equivalence relation (your answer must not depend on my knowing what words like "reflexive", "symmetric", and "transitive" mean: show me that you know what they mean).

   Prove one of them (if you prove more your best effort will count, and you might get a little extra credit).

   If the relation is restricted to the numbers between 1 and 20 inclusive, write out the equivalence classes under this relation (hint: numbers are equivalent if they have the same remainder on division by 5).

7. Do two of the three following proofs (two are induction, one is proof by contradiction). In the math induction proofs, clearly identify the basis step, induction step, and induction hypothesis, and show me where the induction hypothesis is used in your proof.

   If you do all parts your best two will count and extra credit is possible.

   (a) Prove by mathematical induction that the sum of the first n odd numbers is equal to n^2. This sum can be written in the form $\sum_{i=1}^{n}(2i-1)$ or the form $1+3+5+\ldots+(2n-1)$

(b) Prove by mathematical induction that for any natural number n, $n^3 + 5n$ is divisible by 3.

(c) Prove by contradiction that for any sets A and B, $(A - B) \cap (B - A) = \emptyset$. Hint: start by supposing that $(A - B) \cap (B - A)$ is not empty. Then it has an element...